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Self-Tuning control for an extended class of bilinear systems, case of study: nuclear fission mode

Controlador auto-ajustables para una clase extendida de sistemas bilineales, caso de estudio: modelo de fisión nuclear

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Abstract

Taking into consideration that several real systems may be represented in a special class of bilinear systems, in this work, a new algorithm is proposed for the self-tuning control, combining recursive parameters estimation and generalized minimum variance criterion, of an extended and more relaxed class of bilinear systems, where the control action could be presented only in the bilinear term. The closed-loop stability and reference tracking of the proposed self-tuning control is proved using a Lyapunov function. The idea of the proposed algorithm is based on the discrete-time sliding mode control concept. The existence of quasi-sliding regimes for the controlled bilinear system class is also showed. To validate the proposed algorithm, the nuclear fission model is considered as a case of study.

Key words: Bilinear systems, self-tuning control, sliding mode control, nuclear fission.

Resumen

Tomando en cuenta que una gran cantidad de sistemas físicos reales pueden ser representados a través de una clase especial de modelos matemáticos bilineales, en este trabajo se propone un nuevo algoritmo de control auto-ajustable, el cual combina la estimación de los parámetros del control de manera recursiva y el criterio de mínima varianza generalizada, para una clase extendida y más suavizada de sistemas bilineales, donde la acción del control puede estar presente solamente en el término bilineal del modelo. Se demuestra la estabilidad global del sistema en lazo cerrado a través del uso de una función de Lyapunov, además se demuestra el seguimiento de la señal de referencia por parte de la señal de salida. La idea del algoritmo que se propone se basa en el concepto de control por régimen deslizante, por consiguiente se demuestra la existencia de régimen semi-delizante para la clase de sistemas bilineales controlada por el algoritmo propuesto. Como caso de estudio para validar el algoritmo propuesto se considera el modelo de fisión nuclear.

Palabras claves: Sistemas bilineales, controladores auto-ajustables, control por régimen deslizante, fisión nuclear.

1 Introduction

It has been shown under relatively mild conditions that a large class of nonlinear systems can be approximated with arbitrary precision using bilinear models with finite number of coefficients (Brockett, 1976). In addition, many concepts associated with linear systems can be extended to the bilinear case.

Bilinear systems are the simplest class of nonlinear systems and can also be regarded as a practical starting point for the study of other nonlinear systems. However, only a few papers have focused on the stabilization problem of bilinear systems with time delay (Sun y col., 1992).

An important issue associated with the bilinear system model is that of its stability. It is possible to find bounded input signals that can cause the output of almost all bilinear systems to be unbounded. This is probably the main reason why only very limited work on the theory of adaptive bilinear filtering has been done.

Self-tuning control of linear systems has been studied since the seventies. Nevertheless, only a few papers are concerned with the self-tuning control of bilinear systems. An important contribution was given by (Goodwin y col., 1984). They presented an explicit self-tuning controller for bilinear systems; however it lacked of a rigorous stability proof.

The stability of implicit self-tuning control, based on generalized minimum variance criterion for minimum and a class of non-minimum phase linear systems, has been proved by the use of a Lyapunov function in (Patete y col., 2008a; 2008b), and for those systems, it suffices to use linear functions of the data to predict the system output response. In general, it may be desirable, or even necessary, to consider the use of nonlinear functions to get good predictions and hence good control performance.

Sun (Sun y col., 1992) gave a proof for the explicit self-tuning controller of bilinear systems. However, their proof relies on the strong condition of assuming parameters convergence in the closed-loop system, when the projection algorithm is used.

Patete (Patete y col., 2008c; 2011) proved an algorithm which assured control stability and reference tracking for bilinear systems; however the algorithm works under the strong condition where the control variable must appears in a linear term and in the bilinear term of the bilinear system structure, reducing its real application to a small class of bilinear systems.

Motivated by the fact that in practice, a lot of bilinear system structures have the control variable only in the bilinear term, in this work, the class of bilinear systems considered in (Patete y col., 2008c; 2011) is enlarged by relaxing the strong condition on its structure. Also, in this paper, the stability of the implicit self-tuning controllers for the extended class of discrete-time bilinear systems, represented by the input-output relation with unknown parameters, is proved. The algorithm is based on the combination of the generalized minimum variance control and identification of control parameter recursively, although the parameters are not assured to converge to the actual values. Time delay is also taken into consideration.

The paper is organized as follows: in section 2, the generalized minimum variance criterion for the extended class of bilinear systems is given. In section 3 the recursive self-tuning controller parameters estimation, based on the generalized minimum variance criterion, for the extended class of bilinear systems is studied and the main results are given by the theorem which assure closed-loop system stability. The case of study given is analyzed by simulation and is presented in section 4. Some remarks conclude the paper.

2 Generalized minimum variance control for an extended class of bilinear systems

Bilinear systems are a special class of nonlinear systems that are linear in input and linear in state but not jointly linear in both state and input. Specifically, a time invariant single-input and single-output (SISO) bilinear system has a discrete-time form as follows:

$$A(z^{-1})y_k = z^{-d}B(z^{-1})u_k + z^{-d}M(z^{-1})y_k u_k, \tag{1}$$

where,

$$A(z^{-1}) = 1 + a_1 z^{-1} + ... + a_n z^{-n},$$
$$B(z^{-1}) = b_0 + b_1 z^{-1} + ... + b_m z^{-m},$$
$$M(z^{-1}) = m_0 + m_1 z^{-1} + ... + m_\varsigma z^{-\varsigma},\ \ m_0 \neq 0.$$

It is assumed that there are no common factors in $(A(z^{-1}), M(z^{-1}))$, or in $(A(z^{-1}), B(z^{-1}))$ and the time delay d is known. z denotes the time shift operator $z^{-t}y_k = y_{k-t}$. In the Laplace transformation, $z = e^{sT_0}$ where T_0 is the sampling period (for simplicity, and without loss of generality, $T_0 = 1$ is assumed for mathematical equations' derivation). In this section, to derive the nominal control law the polynomials $A(z^{-1})$, $B(z^{-1})$ and $M(z^{-1})$ are assumed to be known.

Remark 1: The extended class of bilinear systems to be considered in this paper is the class where the discrete-time system can be described as in (1), with polynomial $A(z^{-1}) \neq 0$, $d > 0$, $M(z^{-1}) \neq 0$, and the bilinearity is considered only between the output (measured state) and the input variable. Note that polynomial $B(z^{-1})$ could be equal to zero in this structure, which gives a more relaxed structure than the one given in (Patete y col., 2008c; 2011).

If $M(z^{-1}) = 0$, and $b_0 \neq 0$, the described system in (1) is in a linear form and, for this case, the reader may be referred to (Patete y col., 2008c; 2011).

The following notations are introduced:

$$z^{-t}u_k = u_{k-t},$$
$$(z^{-t}u_k)v_k = u_{k-t}v_k,$$
$$z^{-t}u_k v_k = u_{k-t}v_{k-t},$$
$$z^{t}z^{-t}u_k v_k = z^{t}z^{-t}(u_k v_k) = u_k v_k.$$

It was proposed in (Patete y col., 2008c; 2011) an algorithm that assured control stability and reference tracking for the nominal system described in (1), if and only if polynomial $B(z^{-1})$ in (1) is not zero, i.e. $B(z^{-1}) \neq 0$. This is a strong condition on the bilinear system structure. A new algorithm (where $B(z^{-1})$ no necessarily should be not zero) is considered and proved in this paper as follows,

Algorithm 1:

The control objective is to minimize the variance of the linear controlled sliding mode variable s_{k+d}, defined as:

$$s_{k+d} = C(z^{-1})(y_{k+d} - r_{k+d}) + Q(z^{-1})u_k. \tag{2}$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + c_2 z^{-2} + \ldots + c_n z^{-n}, \tag{3}$$

$$Q(z^{-1}) = q_0(1 - z^{-1}), \tag{4}$$

are to be designed, so that the specification written below should be satisfied. The error signal e_k was defined as: $e_k = y_k - r_k$, where r_k is the reference signal. The idea is based on the discrete-time sliding mode control (Furuta, 1990; 1993). The polynomial $C(z^{-1})$ is chosen Schur, and is designed by assigning all characteristic roots inside the unit disk of the z-plane.

To derive the nominal control law, (1) is multiplied by

$E(z^{-1})$, then:

$$E(z^{-1})A(z^{-1})y_k = z^{-d}E(z^{-1})B(z^{-1})u_k + z^{-d}E(z^{-1})M(z^{-1})y_k u_k. \tag{5}$$

Using the Diophantine equation:

$$C(z^{-1}) = A(z^{-1})E(z^{-1}) + z^{-d}F(z^{-1}), \tag{6}$$

where,

$$E(z^{-1}) = e_0 + e_1 z^{-1} + \ldots + e_{d-1} z^{-d+1},$$
$$F(z^{-1}) = f_0 + f_1 z^{-1} + \ldots + f_{n-1} z^{-n+1},$$

(5) is rewritten as:

$$C(z^{-1})y_k - z^{-d}F(z^{-1})y_k = z^{-d}E(z^{-1})B(z^{-1})u_k + z^{-d}E(z^{-1})M(z^{-1})y_k u_k, \tag{7}$$

$$C(z^{-1})y_{k+d} = F(z^{-1})y_k + E(z^{-1})B(z^{-1})u_k + E(z^{-1})M(z^{-1})y_k u_k. \tag{8}$$

Combining (8) and (2), the variable s_{k+d} is:

$$s_{k+d} = G(z^{-1})u_k + F(z^{-1})y_k - C(z^{-1})r_{k+d} + H(z^{-1})y_k u_k, \tag{9}$$

Where $G(z^{-1}) = E(z^{-1})B(z^{-1}) + Q(z^{-1})$ and

$H(z^{-1}) = E(z^{-1})M(z^{-1})$.

Then, the generalized minimum variance control input required to vanish s_{k+d} in (2) is given by:

$$u_k = -\frac{F(z^{-1})y_k - C(z^{-1})r_{k+d}}{G(z^{-1}) + H(z^{-1})y_k}. \tag{10}$$

Remark 2: Note that in this algorithm, the proposed sliding mode surface is linear and gives a nonlinear control law, which is a different algorithm from the one proposed in (Patete y col., 2008c; 2011) where the sliding mode control, s_{k+d}, was given nonlinear and the resultant control law, u_k, was linear.

Remark 3: Polynomial $Q(z^{-1})$ must be designed as in (4) for the reference tracking to be assured (Patete y col., 2008a).

The fact that polynomial $B(z^{-1})$ may be equal to zero enlarge the class of bilinear systems to work with. Several systems may be modeled by a bilinear equation where the control variable only appears in the bilinear term.

2.1 Quasi-sliding regimes for the extended class of controlled bilinear systems

Consider a smooth single-input single-output discrete-time nonlinear system of the form:

$$x_{k+1} = \Psi(x_k, u_k),$$
$$y_k = \Omega(x_k), \tag{12}$$

where x_k is the state vector. The level set,

$$\Omega^{-1}(0) := \{x \in X : \Omega(0) = 0\}, \tag{13}$$

where X contains all x, defines the sliding manifold which is assumed to be sufficiently smooth (Sira-Ramirez, 1991).
Definition 1: The controlled system (11) and (12) is said to exhibit a quasi-sliding motion locally whenever (Sarpturk y col., 1987)

$$y_k(y_{k+d} - y_k) < 0. \tag{14}$$

Definition 1 represents the discrete-time counterpart of the continuous-time sliding mode condition, $y(t)\dot{y}(t) < 0$ and it is also trivially equivalent to have $y_k y_{k+d} < y_k^2$ (Furuta, 1993). Based on Definition 1, the quasi-sliding mode existence for the controlled system in (1) is stated in the following Lemma.

Lemma 1: The controlled discrete-time bilinear system (1), is said to exhibit a quasi-sliding motion locally whenever

$$s_k(s_{k+d} - s_k) < 0. \tag{15}$$

Proof: Using (1), (6) and (2), equation (9) is obtained. Substituting (9) in (15):

$$s_k[G(z^{-1})u_k + F(z^{-1})y_k - C(z^{-1})r_{k+d} + H(z^{-1})y_k u_k - s_k] < 0. \tag{16}$$

Substituting (10) in (16), it is obtained:

$$s_k(-s_k) < 0, \tag{17}$$

$$-s_k^2 < 0. \ \square \tag{18}$$

3 Self-tuning control for the extended class of bilinear systems

In this section, the system in (1) is considered as a system with the same structure having parametric uncertainties. The overall stability of the self-tuning control based on generalized minimum variance criterion for SISO linear systems has been proved in (Patete 2008a), when the system constant parameters are not accurately known by recursive estimation of the controller parameter $F(z^{-1})$ and $G(z^{-1})$, i.e. $\hat{F}(z^{-1})$ and $\hat{G}(z^{-1})$ are estimates of $F(z^{-1})$ and $G(z^{-1})$, under the following assumptions,

Assumptions 1:
1) The order of the system (1) is known.
2) The time delay, d, is known.
3) Polynomial $C(z^{-1})$ is Schur.
4) The considered system (1) with parametric uncertainties is in the class of systems which can be stabilized by the polynomials $Q(z^{-1})$ and $C(z^{-1})$ designed for the nominal system model.
5) The reference signal r_k is bounded, i.e. $|r_k| < \delta$ for all k, where δ is a positive constant.

In this section, the closed-loop stability of self-tuning control for the extended class of bilinear systems, based on generalized minimum variance criterion, is given by the following recursive estimation equations:

In this section, the closed-loop stability of self-tuning control for the extended class of bilinear systems, based on generalized minimum variance criterion, is given by the following recursive estimation equations:

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \frac{\Gamma_{k-1}\phi_{k-d}}{1+\phi_{k-d}^T\Gamma_{k-1}\phi_{k-d}}[s_k + C(z^{-1})r_k - \phi_{k-d}^T\hat{\theta}_{k-1}] \tag{19}$$

And

$$\Gamma_k = \Gamma_{k-1} - \frac{\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\Gamma_{k-1}}{1+\phi_{k-d}^T\Gamma_{k-1}\phi_{k-d}}, \tag{20}$$

Where

$$\phi_k^T = [y_k, ..., y_{k-n+1}, u_k, ..., u_{k-m-d+1}, ..., y_k u_k, y_{k-1}u_{k-1}, ..., y_{k-\varsigma(d-1)}u_{k-\varsigma(d-1)}] \tag{21}$$

is the vector containing measured output and control signal data,

$$\theta^T = [f_0, ..., f_{n-1}, g_0, ..., g_{m+d-1}, ..., h_0, h_1, ..., h_{\varsigma(d-1)}] \tag{22}$$

is the vector containing the controller parameters, and

$$\hat{\theta}^T = [\hat{f}_0, ..., \hat{f}_{n-1}, \hat{g}_0, ..., \hat{g}_{m+d-1}, ..., \hat{h}_0, \hat{h}_1, ..., \hat{h}_{\varsigma(d-1)}] \tag{23}$$

is the estimate of θ.

The controller uses identified parameters as follows:

$$u_k = -\frac{\hat{F}(z^{-1})y_k - C(z^{-1})r_{k+d}}{\hat{G}(z^{-1}) + \hat{H}(z^{-1})y_k}, \tag{24}$$

where $\hat{F}(z^{-1})$, $\hat{G}(z^{-1})$, and $\hat{H}(z^{-1})$ are the estimated of $F(z^{-1})$, $G(z^{-1})$ and , $H(z^{-1})$ respectively.

Theorem 1: Given a positive definite matrix Γ_0 and the initial parameters vector $\hat{\theta}_0$, if the estimate $\hat{\theta}_k$ of the controller (24) satisfies the recursive equations (19) and (20), under the set of Assumptions 1, then the close-loop system, combined by the self-tuning controller (24), (19) and (20) for the bilinear system (1) with parametric uncertainties is stable.

Proof: s_{k+d} is written as:

$$s_{k+d} = \hat{G}(z^{-1})u_k + \hat{F}(z^{-1})y_k - C(z^{-1})r_{k+d} + \phi_k^T\tilde{\theta}_{k-d} \quad (25)$$

where $\tilde{\theta}_k = \theta - \hat{\theta}_k$.

Using the control law (24), (25) is rewritten as:

$$s_{k+d} = \phi_k^T\tilde{\theta}_{k-d}. \quad (26)$$

Consider the candidate Lyapunov function:

$$V_k = \frac{1}{2}s_k^2 + \frac{1}{2}\tilde{\theta}_k^T\Gamma_k^{-1}\tilde{\theta}_k. \quad (27)$$

The time difference of (27) is:

$$\Delta V_k = V_k - V_{k-1}, \quad (28)$$

$$\Delta V_k = \frac{1}{2}s_k^2 - \frac{1}{2}s_{k-1}^2 + \frac{1}{2}\tilde{\theta}_k^T\Gamma_k^{-1}\tilde{\theta}_k - \frac{1}{2}\tilde{\theta}_{k-1}^T\Gamma_{k-1}^{-1}\tilde{\theta}_{k-1}, \quad (29)$$

$$\Delta V_k = -\frac{1}{2}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right)^T\Gamma_{k-1}^{-1}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right) + \frac{1}{2}\tilde{\theta}_k^T\left(\Gamma_k^{-1} + \Gamma_{k-1}^{-1}\right)\tilde{\theta}_k - \frac{1}{2}s_{k-1}^2 + \frac{1}{2}s_k^2 - \tilde{\theta}_k^T\Gamma_{k-1}^{-1}\tilde{\theta}_{k-1}, \quad (30)$$

$$\Delta V_k = -\frac{1}{2}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right)^T\Gamma_{k-1}^{-1}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right) + s_k^2 - \frac{1}{2}s_k^2 - \frac{1}{2}s_{k-1}^2 - \tilde{\theta}_k^T\Gamma_{k-1}^{-1}\tilde{\theta}_{k-1} + \frac{1}{2}\tilde{\theta}_k^T\left(\Gamma_k^{-1} - \Gamma_{k-1}^{-1}\right)\tilde{\theta}_k + \tilde{\theta}_k^T\Gamma_{k-1}^{-1}\tilde{\theta}_{k-1}. \quad (31)$$

From (26), s_k is:

$$s_k^2 = \tilde{\theta}_k^T\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k. \quad (32)$$

Substituting (32) into (31), the following relation is derived

$$\Delta V_k = -\frac{1}{2}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right)^T\Gamma_{k-1}^{-1}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1}\right) - \frac{1}{2}s_{k-1}^2 + \frac{1}{2}\tilde{\theta}_k^T\left(\Gamma_k^{-1} - \Gamma_{k-1}^{-1} - \phi_{k-d}\phi_{k-d}^T\right)\tilde{\theta}_k + \tilde{\theta}_k^T\Gamma_{k-1}^{-1}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1} + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k\right). \quad (33)$$

The term:

$$\frac{1}{2}\tilde{\theta}_k^T\left(\Gamma_k^{-1} - \Gamma_{k-1}^{-1} - \phi_{k-d}\phi_{k-d}^T\right)\tilde{\theta}_k$$

in (33) can be made equal to zero as follows:

$$\Gamma_k^{-1} - \Gamma_{k-1}^{-1} - \phi_{k-d}\phi_{k-d}^T = 0, \quad (34)$$

$$\Gamma_k = \left(\Gamma_{k-1}^{-1} + \phi_{k-d}\phi_{k-d}^T\right)^{-1}, \quad (35)$$

$$\Gamma_k = \Gamma_{k-1} - \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\left(\Gamma_{k-1} + \phi_{k-d}\phi_{k-d}^T\right)^{-1}, \quad (36)$$

that yields (20) by the matrix inversion lemma.

The term:

$$\tilde{\theta}_k^T\Gamma_{k-1}^{-1}\left(\tilde{\theta}_k - \tilde{\theta}_{k-1} + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k\right)$$

in (33) also can be made equal to zero as described below:

$$\tilde{\theta}_k - \tilde{\theta}_{k-1} + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k = 0, \quad (37)$$

$$\tilde{\theta}_k + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_k = \tilde{\theta}_{k-1}, \quad (38)$$

$$\left(I + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\right)\tilde{\theta}_k = \left(I + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\right)\tilde{\theta}_{k-1} - \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\tilde{\theta}_{k-1}, \quad (39)$$

$$\tilde{\theta}_k = \tilde{\theta}_{k-1} + \frac{\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\left(\theta - \hat{\theta}_{k-1}\right)}{1 + \phi_{k-d}\Gamma_{k-1}\phi_{k-d}^T}. \quad (40)$$

From (9):

$$s_k = \phi_{k-d}^T \theta - C(z^{-1}) r_k , \tag{41}$$

thus (19) is derived.

Using the recursive equations (19) and (20) in (33), for $k=1$, the following relation is obtained:

$$V_1 - V_0 = -\frac{1}{2} s_0^2 - \frac{1}{2} \left(\tilde{\theta}_1 - \tilde{\theta}_0 \right)^T \Gamma_0^{-1} \left(\tilde{\theta}_1 - \tilde{\theta}_0 \right) . \tag{42}$$

Initially $\tilde{\theta}_1 - \tilde{\theta}_0 \neq 0$, then $V_1 - V_0 < 0$ which gives that $V_1 < V_0$. For $k=2$,

$$V_2 + \frac{1}{2} s_1^2 + \frac{1}{2} \left(\tilde{\theta}_2 - \tilde{\theta}_1 \right)^T \Gamma_1^{-1} \left(\tilde{\theta}_2 - \tilde{\theta}_1 \right) = V_1 < V_0 . \tag{43}$$

For $k=3$,

$$V_3 + \frac{1}{2} s_2^2 + \frac{1}{2} \left(\tilde{\theta}_3 - \tilde{\theta}_2 \right)^T \Gamma_2^{-1} \left(\tilde{\theta}_3 - \tilde{\theta}_2 \right) = V_2 , \tag{44}$$

using (43) and (44), the following is obtained:

$$V_3 + \frac{1}{2} s_2^2 + \frac{1}{2} s_1^2 + \frac{1}{2} \left(\tilde{\theta}_3 - \tilde{\theta}_2 \right)^T \Gamma_2^{-1} \left(\tilde{\theta}_3 - \tilde{\theta}_2 \right) + \frac{1}{2} \left(\tilde{\theta}_2 - \tilde{\theta}_1 \right)^T \Gamma_1^{-1} \left(\tilde{\theta}_2 - \tilde{\theta}_1 \right) = V_1 < V_0 . \tag{45}$$

Then, for $k=N$, where N is large, the following relation is derived:

$$V_N + \frac{1}{2} \sum_{k=2}^{N} [s_{k-1}^2 + \left(\tilde{\theta}_k - \tilde{\theta}_{k-1} \right)^T \Gamma_{k-1}^{-1} \left(\tilde{\theta}_k - \tilde{\theta}_{k-1} \right)] = V_1 < V_0 , \tag{46}$$

$$V_N + \frac{1}{2} \sum_{k=2}^{N} [s_{k-1}^2 + \left(\tilde{\theta}_k - \tilde{\theta}_{k-1} \right)^T \Gamma_{k-1}^{-1} \left(\tilde{\theta}_k - \tilde{\theta}_{k-1} \right)] < V_0 < \infty . \tag{47}$$

For any $k = N \ (k > 2)$, inequality (47) holds. Equation (47) implies that s_N and $\hat{\theta}_N - \hat{\theta}_{N-1}$ vanish as N approaches infinity, thus ΔV_k is negative semi-definite for all k and the generalized minimum variance is minimized, which proves the overall system stability.

As a result from the above proof, ϕ_k^T is bounded. This means that:

$$\begin{aligned} & y_k < \infty, y_{k-1} < \infty, \ldots, y_{k-n+1} < \infty, \\ & u_k < \infty, u_{k-1} < \infty, \ldots, u_{k-m-d+1} < \infty, \\ & y_k u_k < \infty, \ldots, y_{k-\varsigma(d-1)} u_{k-\varsigma(d-1)} < \infty \end{aligned}$$

are bounded for all k. Furthermore as $k \to \infty$, $s_k \to 0$ and $\left(\tilde{\theta}_k - \tilde{\theta}_{k-1} \right) \to 0$, which means that $\hat{\theta}_k$ goes to a constant value.

The actual value y_k is shown to be bounded as follows:

Multiplying (2) by $B(z^{-1})$,

$$B(z^{-1}) s_{k+d} = B(z^{-1}) C(z^{-1}) y_{k+d} - B(z^{-1}) C(z^{-1}) r_{k+d} + B(z^{-1}) Q(z^{-1}) u_k , \tag{48}$$

$$B(z^{-1}) s_k = B(z^{-1}) C(z^{-1}) y_k - B(z^{-1}) C(z^{-1}) r_k + z^{-d} B(z^{-1}) Q(z^{-1}) u_k , \tag{49}$$

and using (1):

$$B(z^{-1}) s_k = B(z^{-1}) C(z^{-1}) y_k - B(z^{-1}) C(z^{-1}) r_k + A(z^{-1}) Q(z^{-1}) y_k - z^{-d} Q(z^{-1}) M(z^{-1}) y_k u_k , \tag{50}$$

$$y_k = \frac{B(z^{-1})}{T(z^{-1})} s_k + \frac{B(z^{-1}) C(z^{-1})}{T(z^{-1})} r_k + \frac{Q(z^{-1}) M(z^{-1})}{T(z^{-1})} y_{k-d} u_{k-d} , \tag{51}$$

where $T(z^{-1})$ is defined as:

$$T(z^{-1}) = C(z^{-1}) B(z^{-1}) + A(z^{-1}) Q(z^{-1}). \tag{52}$$

The signal s_k was proved to go to zero as $k \to \infty$. The signal r_k is assumed to be bounded for all k and the signal $y_{k-d} u_{k-d}$ was proved to be bounded from the boundeness of vector ϕ_k^T. From the set of Assumptions 1, number 4 means that the closed-loop characteristic polynomial, considering the described plant with parametric uncertainties, in (1), $T(z^{-1})$, is Schur. Thus, y_k in closed-loop is proved to be bounded. Furthermore, the error $e_k = y_k - r_k$ is bounded.

4 Case of study: nuclear fission model

When a fertile nucleus undergoes fission, an average of two or three neutrons are emitted together with nuclear radiation and a relatively large amount of energy. The energy causes rapid motion of fission fragments, which produces heat. The dynamic model follows that of (Mohler, 1991).

The net change in neutron population over one generation by neutron conservation is

$$\frac{dn(t)}{dt} = (K-l)\frac{n(t)}{l}, \tag{53}$$

where K, the average number of first-generation offspring per neutron death, is called the multiplication constant, l is the mean prompt neutron generation time; and all neutrons are produced in t time t □ l. Here l may be a millisecond for thermal reactors, or might be a microsecond for fast reactors. This model assumes that all neutrons are produced promptly. It is common knowledge, however, that a small portion of neutrons are derived from unstable fission products.

Equation (53) for prompt neutrons may be modified to account for delayed neutrons merely by subtracting $\frac{K\beta n}{l}$, and for a neutron source rate σ, by adding σ. Then the additional delayed neutrons emitted by the six precursors cause the rate of neutron change to be

$$\frac{dn(t)}{dt} = \frac{K(1-\beta)-1}{l}n(t) + \sum_{i=1}^{6}\lambda_i c_i(t) + \sigma, \tag{54}$$

where λ_i is the decay constant for the ith group of precursors and c_i is the population of the ith precursors group. It is assumed that delayed neutrons have the same effect on the process as that of prompt fission neutrons. Here source σ is usually a relatively small constant.

The rate of precursor population change equals birth rate minus death rate:

$$\frac{dc_i(t)}{dt} = \frac{dc_i(t)}{dt} = \frac{K\beta_i}{l}n(t) - \lambda_i c_i(t),$$
$$i = 1, 2, \ldots, 6, \tag{55}$$

where β_i is the portion of neutrons generated from the ith precursor.

The total neutrons population is a constant value n_1 if:

$$K = \left(l\sum_{i=1}^{6}\frac{\dot{c}_i(t)}{n_i(t)} \right) + 1,$$

and $\sigma = 0$. At this delayed critical condition, the neutron kinetics, (54) and (55), are in an equilibrium state, $n(t) = n_1$ and $c_i(t) = c_{1i}(t)$, $(i = 1, 2, \ldots, 6)$, if $K = 1$ and

$$c_i(t) = \frac{\beta}{l\lambda_i}n_1.$$

Ordinarily, around the design level the system is operated near delayed critical with k approximately 1 and s negligible. Then the kinetics is approximated by:

$$\frac{dn(t)}{dt} = \frac{\delta k - \beta}{l}n(t) + \sum_{i=1}^{6}\lambda_i c_i(t),$$

$$\frac{dc_i(t)}{dt} = \frac{\beta_i}{l}n(t) - \lambda_i c_i(t), \quad i = 1, 2, \ldots, 6,$$

where the reactivity decay δK is $K-1$.

Frequently, the neutron kinetics is accurately approximated by a single-precursor model of the form:

$$\frac{dn(t)}{dt} = \frac{\delta K - \beta}{l}n(t) + \lambda c(t),$$

$$\frac{dc(t)}{dt} = \frac{\beta}{l}n(t) - \lambda c(t)$$

where λ is an average decay constant for an average precursor of population .

Control of this fission process is affected by means of reactivity δk or multiplication K; either may be utilized as a control variable, depending on which equation is used. With the control signal defined as $u(t) = \delta K$, the neutron fission process is a bilinear system. Then, the model is given by:

$$\frac{dn(t)}{dt} = \frac{u(t) - \beta}{l}n(t) + \lambda c(t), \tag{56}$$

$$\frac{dc(t)}{dt} = \frac{\beta}{l}n(t) - \lambda c(t), \tag{57}$$

where the output variable is $y(t) = n(t)$.

In order to use the proposed self-tuning controller, the equations (56) and (57) are discretized using the first order

Euler approximation. Then, the dicretized system structure is given by:

$$(1+a_1z^{-1}+a_2z^{-2})y_{kT_0}=z^{-1}(m_0+m_1z^{-1})y_{kT_0}u_{kT_0}, \quad (58)$$

Where:

$$a_1=\frac{\beta}{l}T_0+\lambda T_0-2, \quad a_2=1-\lambda T_0-\frac{\beta}{l}T_0,$$
$$m_0=\frac{T_0}{l}, \quad m_1=\frac{T_0^2\lambda}{l}-\frac{T_0}{l}.$$

The sampling time T_0 is chosen as 0.001 s. The parameter λ is a parameter that may change, $0.1\le\lambda\le 0.05$, depending if the power is increasing, decreasing or the system is in steady state.

To compute the nominal controller, $\lambda=0.077$ s-1 is assumed, the parameter l is chosen as $l=7\times10^{-5}$ s, and the constants β is $\beta=0.0065$.

Then, the nominal controller is designed as follows:

Polynomials $C(z^{-1})$ and $Q(z^{-1})$ are chosen as:

$$C(z^{-1})=1+0.25z^{-1}+0.3z^{-2}+0.2z^{-3}, \quad (59)$$

$$Q(z^{-1})=0.02(1-z^{-1}). \quad (60)$$

Using the information in (58), (59), (60) and solving the Diophantine equation (6), following the algorithm proposed in (Ogata, 1995), then the polynomials for the nominal control law (10) are obtained as:

$$E(z^{-1})=1-0.220491z^{-1},$$
$$F(z^{-1})=1.93657-0.186575z^{-1},$$
$$G(z^{-1})=Q(z^{-1}),$$
$$H(z^{-1})=14.2857-11.1347z^{-1}-3.14963z^{-2}. \quad (61)$$

Polynomials $F(z^{-1})$, $G(z^{-1})$ and $H(z^{-1})$ give the initials values for $\hat{F}(z^{-1})$, $\hat{G}(z^{-1})$, and $\hat{H}(z^{-1})$, respectively, in (24). The variables initial conditions are: $n(0)=0$ and $c(0)=2$. The reference signal is set to cero, because the goal is to vanish the delayed neutrons after the nuclear fission occurs. Fig. 1 shows the output response of the system (58), using the proposed self-tuning algorithm ((24), (19) and (20)) and no parametric uncertainties is considered in this case ($\lambda=0.077$ s-1).

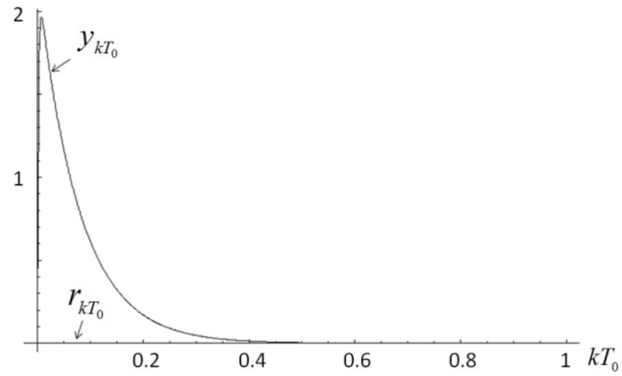

Fig. 1. Output response of system (58) using the proposed algorithm, no parametric uncertainties is considered in this case, $\lambda=0.077\ s^{-1}$.

As shown through the simulations results, even though polynomial $B(z^{-1})$ is equal to zero, the overall stability and tracking reference are satisfied, which allows to deal with a longer class of bilinear systems.

Consider now a system having parameter uncertainties in parameter λ, (to show the simulation results $\lambda=0.09$ s^{-1} is considered), then (58) presents parametric uncertainties. Simulation result for this case is shown in Fig. 2; Fig. 3 and Fig. 4 shown the control dynamics u_{kT_0} and the quasi-sliding motion (15), respectively, for this case when $\lambda=0.09$ s^{-1}. It is shown that the proposed self-tuning algorithm ((24), (19) and (20)) exhibits good performance and the reference is followed in steady state.

5 Conclusions

The extended class of bilinear systems considered in this work may have the bilinearity only between the output (measured state) and the input variable, which enlarge the class of bilinear system to deal with. The proposed self-tuning approach enables controller parameters to be estimated. The closed-loop stability of the implicit self-tuning control for the extended class of bilinear systems was proved. Control stability and reference tracking are shown to be assured. The given algorithm is based on the idea of the sliding mode control concept; because of this the existence of quasi-sliding regimes for the controlled discrete-time bilinear system was also proved. The validity of the proposed algorithm was also demonstrated through a study case: nuclear fission model. As a future work, the proposed algorithm is to be applied to a real bilinear plant where the control signal appears only in the bilinear term.

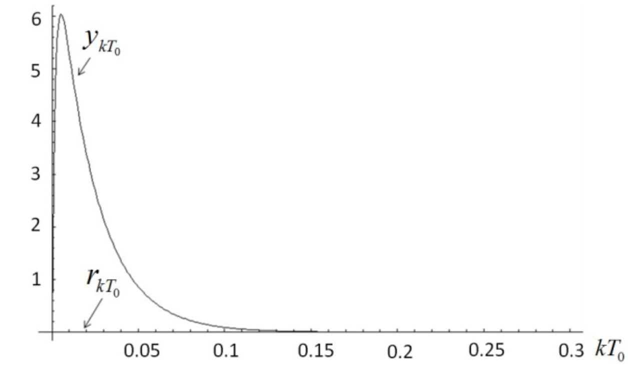

Fig. 2. Output response of system (58), with parameter uncertainties, $\lambda = 0.09$ s^{-1}.

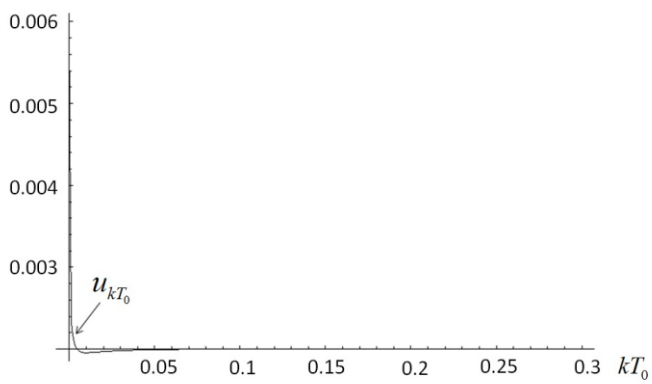

Fig. 3. Control law (24) of system (58) with parameter uncertainties, $\lambda = 0.09$ s^{-1}.

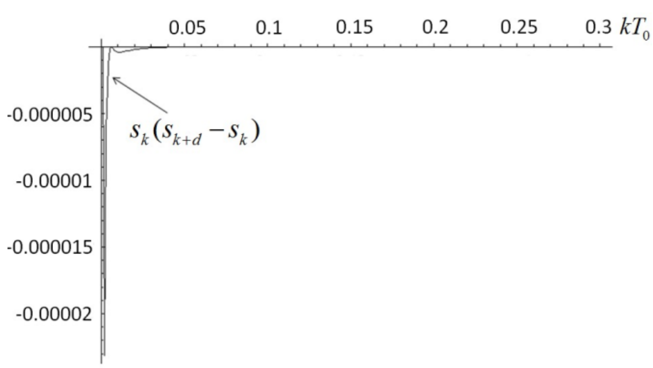

Fig. 4. Quasi-sliding motion (15) of system (58) with parameter uncertainties, $\lambda = 0.09$ s^{-1}

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