Modeling, simulation and animation of an arc reflex system

Modelado, simulación y animación de un sistema arco reflejo

Patete, Ana*, Márquez, Pedro Rodríguez-Millán, Jesús
Departamento de Sistemas de Control, Facultad de Ingeniería, ULA.
Mérida 5101, Venezuela
*apatete@ula.ve

Resumen

The study of biological and physiological systems using mathematical modeling and simulation techniques represents an emerging tendency within engineering and medicine. In this paper, a model for the arc reflex is obtained as example of a physiological control system. To this purpose, a model for each of the associated systems is proposed; muscle spindle as sensor, muscle as actuator, and the nerve structure of the arc reflex as signal processor and controller. Also, the interaction and combined dynamics of these elements is studied as part of a stretch reflex global system. Furthermore, a computational mimic is developed in order to show the system behavior via intuitive graphic animations that depicts the system as it is seen in reality. The mimic's movement is based on the dynamics of the obtained mathematical model.

Palabras clave: Arc reflex, computational mimic, mathematical model, muscle contraction, muscle spindle.

Abstract

El estudio de sistemas biofísicos y fisiológicos a través de las técnicas de modelado y simulación representa una tendencia en auge dentro de la ingeniería y la medicina. En este trabajo, se obtiene un modelo matemático para un arco reflejo como ejemplo de un sistema fisiológico de control y procesamiento de señales. Para esto se plantea un modelo para cada uno de los sistemas asociados; el huso muscular como sensor, el músculo esquelético como actuador y la estructura nerviosa del arco reflejo como procesador de señales y controlador. Se estudia además la interacción y dinámica combinada de estos sistemas y otros elementos con el fin de obtener un modelo que acople todos los componentes. Además, se desarrolla un mímico computacional que permite la visualización del comportamiento del sistema en estudio, por medio de animaciones gráficas intuitivas que se asemejen a la realidad del sistema. El movimiento del mímico es basado en la dinámica del modelo matemático obtenido.

Key words: Arco reflejo, mímico computacional, modelo matemático, contracción muscular, huso muscular.

1 Introducción

Biological systems have been present in nature since millions of years, through which they have evolved to the point of being optimal. Engineering, on the other hand, counts only with a few thousand years of development, which is why there is a tendency on studying these processes to the purpose of applying techniques and structures of biological systems to other kinds of systems.

For engineers, to find a model that represents the nature and behavior of a real system is the first step to develop some application to that system. There are several method and techniques to represent a real system by a model, such as mathematical differential equations (Beltrami 1997, Ogata 2004), dynamic system modeling (Ramírez y col., 2011), average model for switched dynamics (Gómez y col., 2012), using Petri net theory (Aguirre y col., 2012, 2013), to name some. However it is not easy to represent some real models by mathematical equations, and it is more complicated when some variable are not possible to measure directly, such as some signal in our biological system.

One application of engineering techniques, such as modeling and simulating, to biological systems emerges when we take the arc reflex as case of study. The arc reflex is a nerve structure that supports anatomically the involuntary neuromuscular responses produced by effect of stimulus of different kinds. The stretch reflex or myotatic reflex is a particular case of an arc reflex system, where a muscle stretching produces, as response, a contraction that opposes this stretch. The most simple and known stretch reflex is the patellar reflex (Ermentrout y col., 2010).

In recent years, there have been several attempts to capture the dynamics and essential characteristics of stretch reflexes and other physiological systems through mathematical models, all of them with different motivations and points of view. One of the first models for the stretch reflex was presented by (Milhorn, 1966), being also pioneer of applying control theory on biological systems. Later, (Perreault y col., 2000) developed a model for the stretch reflex based in system identification with non-invasive methods. As a continuation of previous work (Santillán y col., 2003) presented a model for the stretch reflex of the cat soleus muscle, studying the muscle response under different levels of activation. Other researchers like (Shadmehr y col., 2005) studied simple models for the muscle and muscle spindle responses using the mechanical analogy of (Hill, 1938), and used identification techniques to validate them experimentally. Moreover, (Ermentrout y col., 2010) studied and detailed in their book the models obtained by (Hodgkin y col., 1952) for resting and action potentials in neuron membranes, which establish analogies between the physiological systems and electrical circuits.

Besides all the previous work on modeling and simulating the stretch reflex and muscle behavior, there has been works on the creation of computational mimics or animations to recreate physical systems as they are seen in reality. Some examples of them are the works of (Rodríguez-Millán y col., 2002, 2003a, 2003b, 2005a) creating an integrated environment of symbolic-graphic-numeric calculation and a library for the computational package Mathematica®, used to build mimics of simple physical systems which exhibit the effect of control laws in discrete-time.

Along this paper, we present at first in section 2 a mathematical modeling of the arc reflex system, starting from an approximation of a patellar reflex response with an analogous pendulum model, then modeling each component; muscle spindle as sensor, muscle as actuator, and neurons as signal processor and controller, studying at last the relations between them and the arc reflex system as a whole. Also, in section 3, we make simulations based on experimental data of previous works from other researchers

(Shadmehr y col., 2005) to show the response of the stretch reflex system modeled. Later, computational mimics are created in section 4 permitting to visualize the dynamics of the stretch reflex model and the relationship between this behavior and the anatomical structure of the system in an intuitive animated fashion. Finally, conclusions of the work done are presented in section 5.

2 Modeling of the system

In this section, the dynamical and physical properties of the stretch reflex, muscle spindle and muscle are studied, and mathematical models are obtain in order to represent the behavior of these systems. Also, we establish relations between each one of these elements within the global arc reflex system.

2.1 Pendulum Model

The most known and simply structured arc reflex system is the patellar reflex. As a first approximation of a mathematical model for this system, a physical pendulum model is proposed analogous to the reflex with regard to the dynamics of the angle formed between the pendulum (or the extremity in the real system) and the vertical axis, without taking into account physiological properties. In this model shown in Fig. 1; θ is the angular position, the input τ corresponds to a force that represents the tension produced by the agonist muscle contraction, the friction constant in the rotation axis $b1$ corresponds to the analogous resistance opposed by the antagonist muscle, m is the pendulum mass, and l denotes the length between the rotation axis and the center of mass of the pendulum. The model obtained for the pendulum system (Ogata, 2004) is represented by differential equation (1):

$$J\,\ddot{\theta}(t) + b1\,\dot{\theta}(t) + m\,l\,\mathrm{g}\sin(\theta(t)) = \tau(t), \qquad (1)$$

where J denotes the moment of inertia with respect to the rotation axis, which corresponds to $J = ml^2$ approaching the pendulum to a concentrated mass, and g represents gravity acceleration.

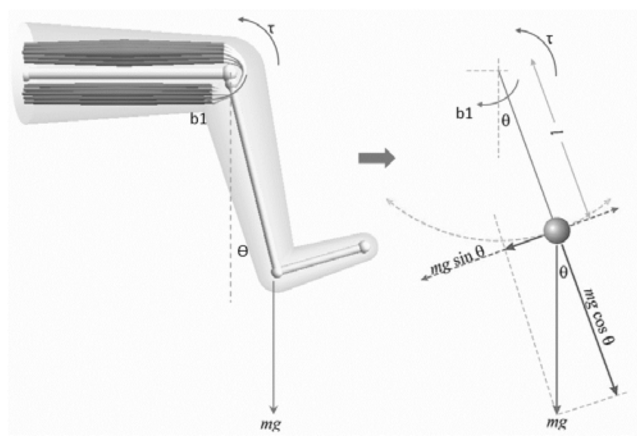

Fig. 1. Physical pendulum diagram, analogous to the patellar réflex

For the posterior development of animations it is necessary to obtain the discrete-time model of non-linear system (1). Thus, we use Euler's method of discretization for non-linear systems proposed by (Rodríguez-Millán y col., 2005a, 2005b) to get the model of equation (2).

$$
\begin{aligned}
x_1(k+1) &= T_0\, x_2(k) + x_1(k),\\
x_2(k+1) &= -\frac{T_0\, m\, l\, g}{J}\sin(x_1(k))\\
&\quad + \left(1 - \frac{T_0\, b1}{J}\right) x_2(k) + \frac{T_0}{J}\tau(k),\\
y(k) &= x_1(k),
\end{aligned}
\tag{2}
$$

where for this particular model $x_1 = \theta$, $x_2 = \dot{\theta}$, $y = \theta$ were assumed and, sampling rate $T_0 = 0.01$ s was chosen to approximate the discrete-time dynamics to the continuous-time response, which showed good approximation.

Using model (2) and parameter values of Table 1, simulations are presented in Fig. 2 and Fig. 3, for a pulse input of magnitude 10 kg and duration 0.5 s starting in $t = 0$ s, analogous to the force produced by the muscle contraction during a stretch reflex. Fig. 2 shows the applied input and in Fig. 3 the response exhibits a quick and slight increment in the value of the angle, to later return in a subamortiguated fashion to the equilibrium resting point $x_{1e} = x_{1e} = \tau = 0$. These accords with the slight extension of the extremity in comparison with patellar reflex response.

Tabla 1. Parameters for pendulum model (2).

Parameters	Value
m	10 kg
l	0.25 m
$b1$	1.5 kg.s.m^{-1}
g	9.8 m.s^{-2}

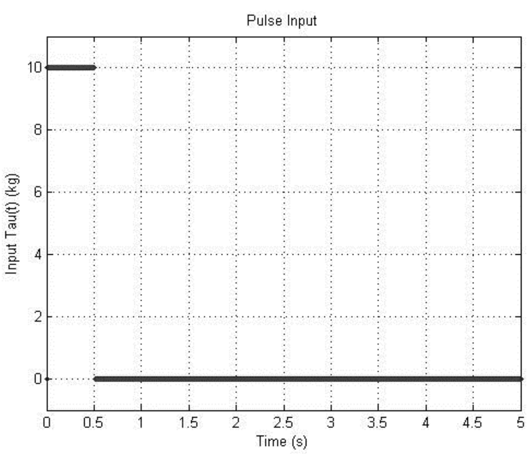

Fig. 2. Pulse input for the pendulum model (2).

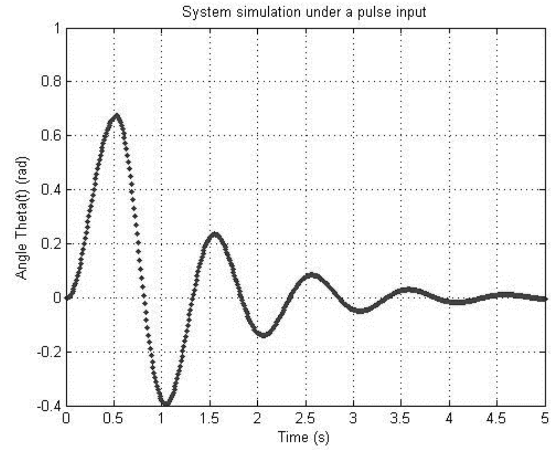

Fig. 3. Pendulum model (2) response

2.2 Muscle Model

Now we study the dynamics of the arc reflex system at a more precise level modeling muscle contractions that produce tension to move the pendulum system. One classical model for the muscle is that proposed by (Hill, 1938), which establishes a mechanical system analogous to the muscle behavior shown in Fig. 4. Muscle produces two types of force, active and passive (Shadmehr y col., 2005). Active force is produced by effect of the contractile mechanism within the sarcomere when muscle fibers are excited via electric stimuli; action potentials from efferent neurons. Passive force is due to intervention of non-contractile elements in muscle, which have elastic or viscous properties. In (Hill, 1938) mechanical model, passive elements are represented by; a spring like elastic element K_S attached in series that represents tendons, another spring like element K_P attached in parallel that performs as membranes and other elastic structures in muscle, and a viscous element modeled as a shock absorber b that introduces dynamics depending on contraction velocity. Also, the input A stands for the active force, the input x represents muscle's length change or one end's displacement considering the other end fixed, and x_c denotes the length change of the contractile element.

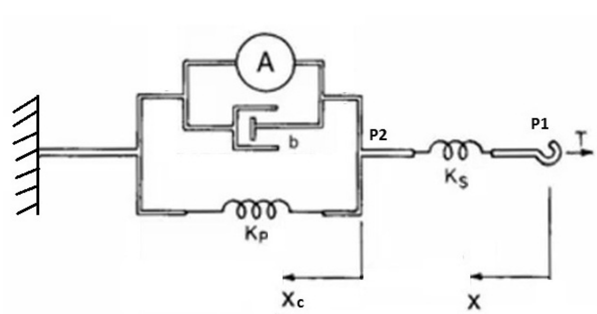

Fig. 4. Hill's muscle model, modified by (Shadmehr, 2005).

Modeling P_1 and P_2 points, according to Newton's second law and assuming these points have no appreciable mass, we obtain a model represented by a differential equation (3) as follows:

$$\dot{T}(t) = -\frac{K_s + K_p}{b} T(t) + \frac{K_s K_p}{b} x(t) + K_s \dot{x}(t) + \frac{K_s}{b} A(t), \tag{3}$$

where $T(t)$ stands for tension developed in the muscle, K_s and K_p are spring constants, and b denotes the shock absorber constant. This model can be rewritten in observable canonic form (Ogata, 1998) as it is shown in (4).

$$\dot{\omega}(t) = -\frac{K_s + K_p}{b} \omega(t) + \begin{bmatrix} -\frac{K_s^2}{b} & \frac{K_s}{b} \end{bmatrix} \begin{bmatrix} x(t) \\ A(t) \end{bmatrix},$$
$$T(t) = \omega(t) + \begin{bmatrix} K_s & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ A(t) \end{bmatrix}. \tag{4}$$

The active force $A(t)$ of the contractile element is given by the response of that element to an impulse train $u(t)$ from central nervous system. Hence, it can be written as (5):

$$A(t) = h(t - t_1) + h(t - t_2) + h(t - t_3) + \ldots, \tag{5}$$

where t_i for $i = 1, 2, 3, \ldots$ are time instants where an input impulse is produced and $h(t)$ represent the impulsive response of the contractile element. This impulsive response corresponds to the sarcomere model, or muscle's contractile element, and may be obtained from experimental data as shown in (Shadmehr y col., 2005). Therefore, the impulsive response is represented by an exponential function of the form (6):

$$h(t) = c_1 e^{-\frac{1}{\alpha 1} t} - c_2 e^{-\frac{1}{\alpha 2} t}, \tag{6}$$

being c_1, c_2, $\alpha 1$ and $\alpha 2$ constant parameters taken from an identification model (Shadmehr y col., 2005).

There are different kinds of contraction that muscle can achieve, isometric and isotonic. Isometric contraction is given when muscle is stimulated electrically by action potentials (impulse trains) but it does not shorten because its ends are fixed, then producing tension. A simulation of isometric contraction is shown using parameter values from Table 2 taken from (Shadmehr y col., 2005) where they have been obtained experimentally.

Tabla 2. Parameters for muscle model (4)

Parameters	Value
K_s	136 g.cm^{-1}
K_p	75 g.cm^{-1}
b	50 g.s.cm^{-1}
c_1	48144
c_2	45845
α_1	0.0326
α_2	0.034

In Fig 5, it can be observed the active force produced by the contractile element under a 20 Hz impulse train, and in Fig. 6 the tension developed in muscle for action of active force and passive elements. Tension developed during each contraction start to sum when frequency is high enough achieving what it is called a sustained contraction or tetanus. In the real system, some saturation values exist corresponding to maximum tension of which muscle is capable, and fatigue.

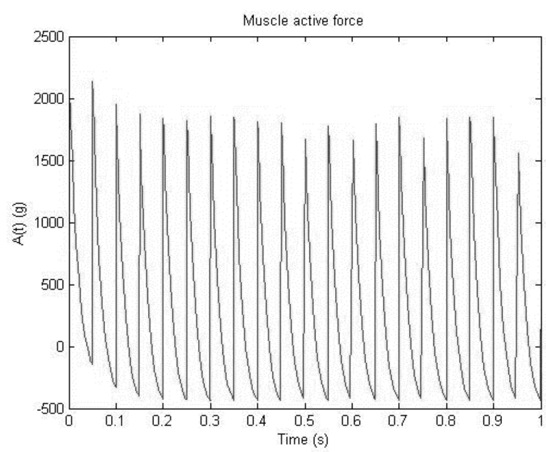

Fig. 5. Active force produced by the contractile element under a 20 Hz impulse train.

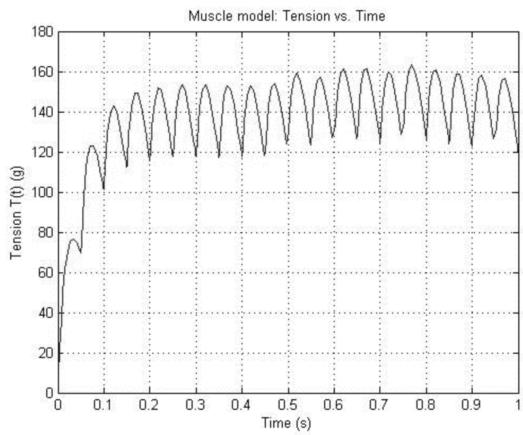

Fig. 6. Muscle response during isometric contraction

The other kind of muscle contraction is isotonic; it corresponds to electrical stimuli and muscle shortening by effect of the contraction. In Fig. 7, a simulation is performed for a 20 Hz impulse train input and a 1 cm shortening during 1 s. In Fig. 8 it can be seen three stages of muscle contraction: isometric contraction reaching tetanus when the muscle is first electrically stimulated, tension drop during isotonic contraction when muscle shortens, and tension increase when muscle returns to resting length.

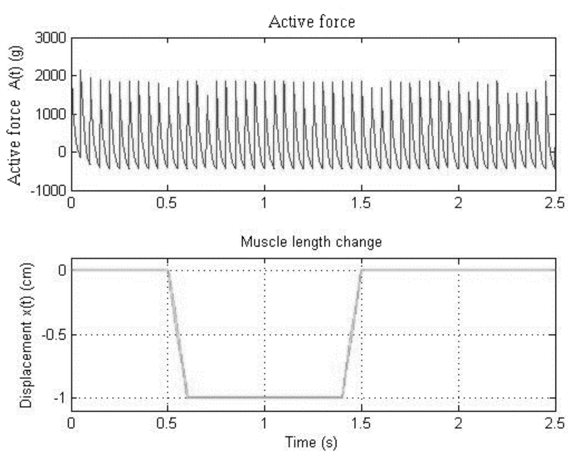

Fig. 7. Impulse train input, 20 Hz, and a 1 cm shortening during 1 s.

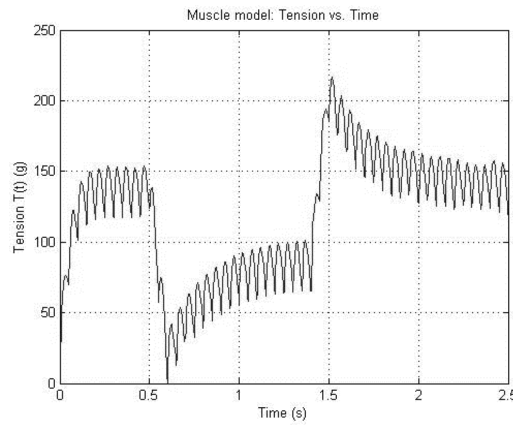

Fig. 8. Muscle during isotonic contraction

Likewise pendulum model, muscle's model is also discretizised using in this case Isermann's exact discretization method (Isermann, 1989), from which (7) is obtained as follows:

$$\begin{aligned} \omega(k+1) &= e^{-\alpha T_0}\omega(k) + \begin{bmatrix} B_1 & B_2 \end{bmatrix}\begin{bmatrix} x(k) \\ A(k) \end{bmatrix}, \\ T(k) &= \omega(k) + \begin{bmatrix} K_s & 0 \end{bmatrix}\begin{bmatrix} x(k) \\ A(k) \end{bmatrix}, \end{aligned} \qquad (7)$$

where:

$$B_1 = -\frac{K_s^2(e^{\alpha T_0}-1)}{(K_p+K_s)e^{\alpha T_0}},$$

$$B_2 = \frac{K_s(e^{\alpha T_0}-1)}{(K_p+K_s)e^{\alpha T_0}},$$

$$\alpha = \frac{K_p+K_s}{b},$$

and T_0 is sampling rate chosen for this model. This discrete-time model will be used further in sections 3 and 4.

2.3 Muscle Spindle Model

Within the arc reflex system, muscle spindle corresponds to the sensor which translates stimuli into an impulse train transmitted via afferent fibers. Muscle spindle's intrafusal fiber behaves in a similar way to muscle fibers; hence, Hill's (Hill 1938) mechanical model can be used as well to represent the dynamics of muscle spindle's tension as shown in Fig. 9. Muscle spindle has two regions, polar and nuclear. Polar region corresponds to contractile element in the mechanical model, and it is inervated by secondary or group II afferents. Also, this region receives electric impulses from central neural system via a gamma-motoneuron in the same way as muscle receives impulses from an alpha-motoneuron. Central or nuclear bag region, on the other hand, has no contractile elements and corresponds to the elastic series element of Hill's model. This region is inervated by primary or group Ia afferents. When muscle spindle is stretched, length change of these regions is translated by afferents into changes in impulse fire rates (Shadmehr 2005).

Since we are using the same mechanical model as for muscle fibers, mathematical model for tension in muscle spindle is similar and shown in (8).

$$\dot{T}_h(t) = -\frac{K_{sh}+K_{ph}}{b_h}T_h(t) + \frac{K_{sh}\,K_{ph}}{b_h}x(t) + K_{sh}\,\dot{x}(t) + \frac{K_{sh}}{b_h}g(t). \tag{8}$$

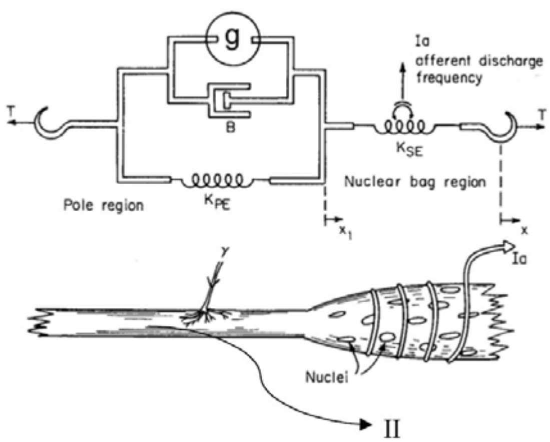

Fig. 9. Muscle spindle model (Hill, 1938).

Assuming that afferent fibers respond linearly to length changes x and x_1, it may be obtained a relation for impulse fire rate in primary ($S_{Ia}(t)$) and secondary afferents ($S_{II}(t)$) as shown in (9) where a stands for a constant value that establishes this linear relation. Even though this response is not quite as linear in reality, the model works as a good approximation (Shadmehr, 2005).

$$\begin{aligned} S_{Ia}(t) &= a(x-x_1), \\ S_{II}(t) &= a\,x_1. \end{aligned} \tag{9}$$

Then, taking into account some relations from the mechanical model and reorganizing equations in the observable canonic form of state space, we obtain muscle spindle model as in (10).

$$\begin{aligned} \dot{\omega}_h(t) &= -\frac{K_{sh}+K_{ph}}{b_h}\omega_h(t) + \begin{bmatrix} -\frac{K_{sh}^2}{b_h} & \frac{K_{sh}}{b_h} \end{bmatrix}\begin{bmatrix} x(t) \\ g(t) \end{bmatrix}, \\ \begin{bmatrix} S_{Ia}(t) \\ S_{II}(t) \end{bmatrix} &= \begin{bmatrix} \frac{a}{K_{sh}} \\ -\frac{a}{K_{sh}} \end{bmatrix}\omega_h(t) + \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ g(t) \end{bmatrix}. \end{aligned} \tag{10}$$

In order to visualize the response of muscle spindle afferents to length change, in Fig. 10 a simulation is given for a 0.5 cm stretch. Parameter values for muscle spindle model are specified in Table 3, which are taken from experimental data of (Shadmehr, 2005). In Fig. 11 we can observe that muscle spindle afferents respond to length change with an increase in impulse fire rate. Primary or Ia afferents respond rapidly during the dynamic phase of stretching, responding to the velocity of the stretch. On the contrary, secondary or II afferents respond increasing gradually the impulse fire rate to a steady value while muscle is still stretched, showing a response to the magnitude of the stretch.

Tabla 3. Parameters for muscle spindle model.

Parameters	Value
K_{sh}	35 g.cm^{-1}
K_{ph}	5 g.cm^{-1}
b_h	10 g.s.cm^{-1}
a	220 Hz.cm^{-1}

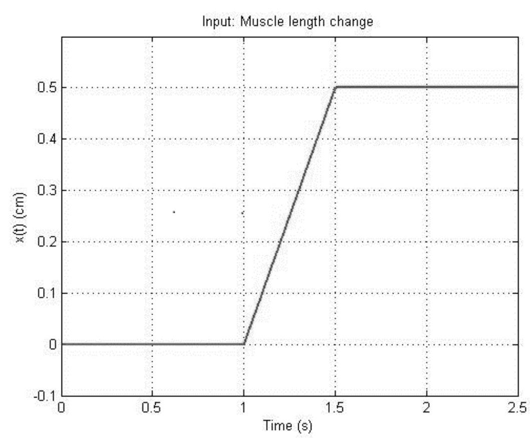

Fig. 10. Stretch of 0.5 cm.

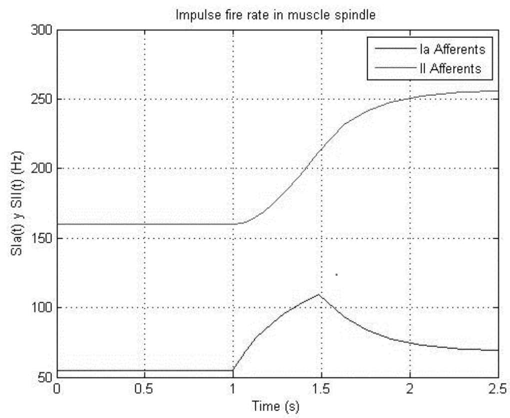

Fig. 11. Muscle spindle model during stretch.

For a time-discrete model, since spindle's tension model is the same as for muscle, muscle spindle model is that shown in (11):

$$\omega_h(k+1) = e^{-\beta T_0}\omega_h(k) + \begin{bmatrix} B_{h1} & B_{h2} \end{bmatrix}\begin{bmatrix} x(k) \\ g(k) \end{bmatrix},$$
$$\begin{bmatrix} S_{Ia}(k) \\ S_{II}(k) \end{bmatrix} = \begin{bmatrix} \dfrac{a}{K_{sh}} \\ -\dfrac{a}{K_{sh}} \end{bmatrix}\omega_h(k) + \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x(k) \\ g(k) \end{bmatrix}, \quad (11)$$

where:

$$B_{h1} = -\frac{K_{sh}^2(e^{\beta T_0}-1)}{(K_{ph}+K_{sh})e^{\beta T_0}},$$
$$B_2 = \frac{K_{sh}(e^{\beta T_0}-1)}{(K_{ph}+K_{sh})e^{\beta T_0}},$$
$$\beta = \frac{K_{ph}+K_{sh}}{b_h},$$

and T_0 denotes the sampling rate chosen for this model. This model will be used for simulation and animation in sections 3 and 4.

2.4 Arc Reflex Model

Once obtained mathematical models for the elementary subsystems of arc reflex, it is possible to establish some relation between them and add new signal conditioner elements, to the purpose of finding a global arc reflex system. In Fig. 12, a block diagram scheme shows the interaction between the studied models and their input and output variables. Since arc reflexes stand for involuntary responses, gamma-motoneurons activation is not taken into account letting muscle spindle model with only one input $x(t)$, which is the global system input. Additionally, pendulum model is attached to the muscle tension output ($T(t)$) through a signal conditioner gain (K). This is made in order to introduce the effect of muscle contraction, due to the arc reflex mechanism, to the movement of an extremity as it occurs in the real system, particularizing the model for a patellar reflex response.

Alpha motoneurons behave as a system controller, they take sensed signals $S_{Ia}(t)$ and $S_{II}(t)$ coming from muscle spindle afferents, and produce a control signal transmitted in an impulse train fashion to the muscle. This controller action is made through the summation of action potentials received via synaptic links with afferent neurons and central nervous system. When a neuron receives an impulse train, a weighted sum is made according to the distance between synapsis and axon's first section (integration center) (Ermentrout y col., 2010). In literature, there are some specific models for action potential generations, like (Hodgkin y col., 1952), a classic and very detailed model for neuron's membrane potentials. However, this model is quite complicated and requires many parameter values that in most of cases are unknown. Therefore, for the case of study, motoneurons behavior is approximated, regarding action potential summation, by means of the relation in (12). In this relation, impulse fire rate of alpha motoneuron $u(t)$ is given by the weighted sum of the changes in impulse frequency $S_{Ia}(t)$ and $S_{II}(t)$ of muscle spindle afferents:

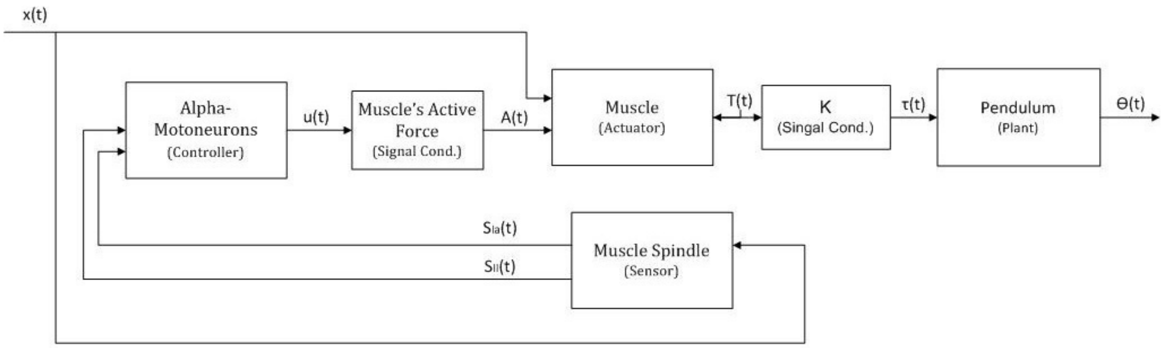

Fig. 12. Block diagram of the arc reflex system.

$$u(t) = a_1(S_{Ia}(t) - S_{Ia}(0)) + a_2(S_{II}(t) - S_{II}(0)), \qquad (12)$$

where a_1 and a_2 are constant values that weigh input signal's frequencies. In Table 4, values for motoneuron model parameters and K gain chosen for simulations are given.

Tabla 4. Parameters for the arc reflex model.

Parameters	Value
a_1	1
a_2	1
K	1/7 kg.g^{-1}

3 Simulation Results

In this section, simulation results are given for the global arc reflex system, using all discrete-time models from section 2. According to parameter values, the sampling rate chosen for the combined discrete-time model is $T_0 = 0.001$ s. The objective of this simulation is to show the response of the whole combined system, to an input analogous to the muscle stretch produced by effect of patellar tendon percussion made to produce a patellar reflex response. Then, the input corresponds to a muscle stretch $x(t)$ of 0.1 cm during 0.1 cm, similar to which occurs when patellar tendon is percussed. In order to really appreciate the dynamics of each one of the involved variables, simulation graphics for all of them are presented. In Fig. 13, we can observe the system's input, and in Fig. 14 the response given by the muscle spindle, which present a small increment in impulse frequency when stimulus is applied.

In Fig. 15 and Fig. 16, the alpha-motoneurons activation and alpha-motoneurons impulse train respectively responses are shown. This response, according to the proposed model, is an increment in impulse fire rate while stimulus is held, returning later to the resting value. Then, in Fig. 17 it may be visualized the active force and in Fig. 18 the tension developed in muscle by effect of alpha-motoneurons activation. Finally, in Fig. 19, pendulum model response is presented by effect of the tension developed in muscle, previously conditioned by gain K. In this graphic it can be seen a pendulum angle response similar to which occurs in the analogous extremity extension by effect of patellar reflex.

Through the use of simulation techniques, we can observe that the response of the studied model resemble the real behavior that stretch reflex system would have, in a living organism, even though there exist certain approximations and simplifications that make the combined system more manageable.

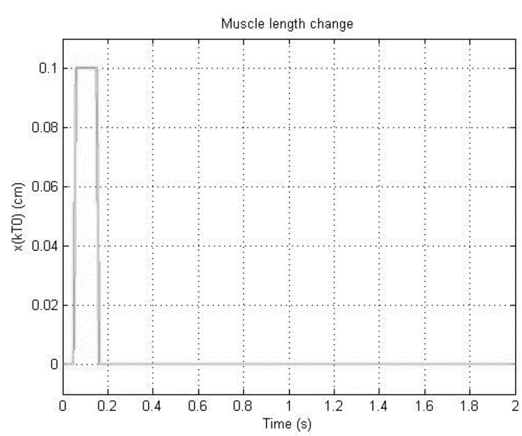

Fig. 13. System's input for the arc reflex model.

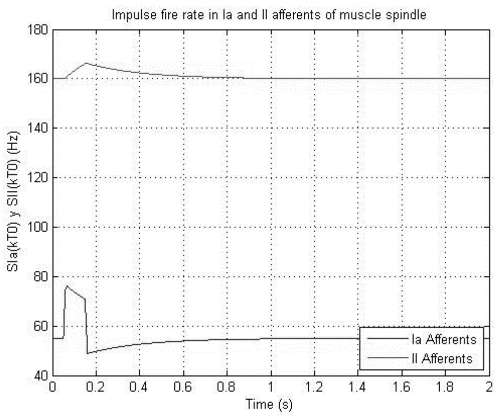

Fig. 14. Muscle spindle response for the arc reflex model

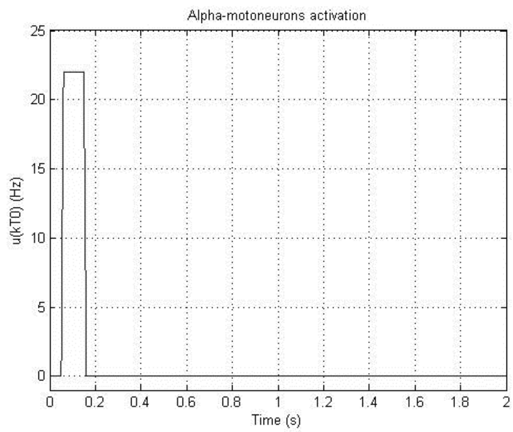

Fig. 15. Alpha-motoneurons response for the arc reflex model

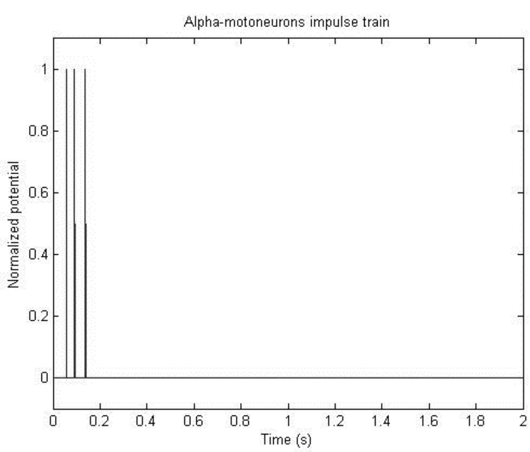

Fig. 16. Action potentials for the arc reflex model.

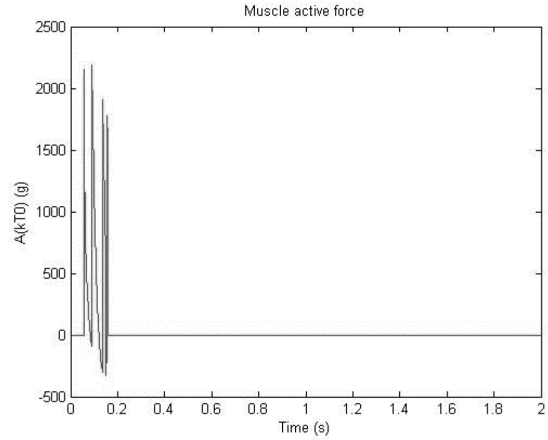

Fig. 17. Muscle’s active force for the arc reflex model

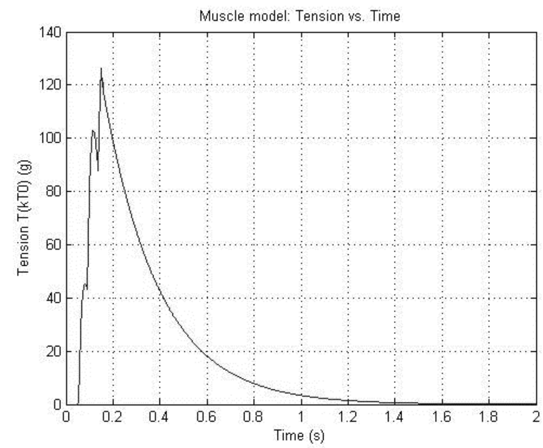

Fig. 18. Tension response for the arc reflex model.

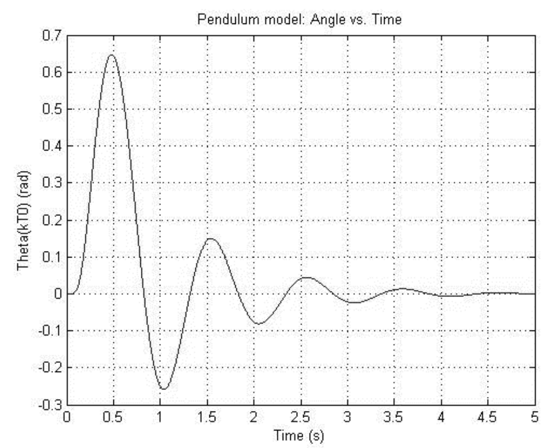

Fig. 19. Pendulum model response

4 System Animation

Along this paper arc reflex models have been analyzed, studied and their behavior shown using mathematical models and traditional numeric graphics. One application of these obtained models can be given when using animation to show their dynamics, permitting to visualize it in a simple intuitive fashion. Computational mimics are graphic animations that represent physical systems as they are seen in reality, without abstracting them into variables and equations representation as it is made for mathematical analysis, though taking mathematical models and simulations into account for their creation. To the purpose of developing computational mimics for the arc reflex system Mathematica® is chosen. This is a software that offers a variety of solutions on dynamic and interactive animation applications, based in graphic elements and numeric calculations.

The computational mimic development process is, according to (Rodríguez-Millán y col., 2002, 2003a, 2003b), given by the following stages: system decomposition into simple graphic elements, system assembly from these graphic elements, creating a sequence of images which represent the system state for each sample from the model simulation, mimic animation using the images sequence as a film reproduction of frames. All of these stages can be made using Mathematica®'s commands like *Table* as a repetition structure for creating the sequence of images, and *ListAnimate* to animate the frames in an interactive structure with reproduction buttons.

4.1 Development of Mimics for the Arc Reflex System

For the stretch reflex system studied in this paper, four different computational mimics are presented: patellar reflex response (Fig. 20), muscle (Fig. 21), muscle spindle (Fig. 22), and arc reflex mechanism (Fig. 23). These are made using the procedure explained before and the simulations performed for models given in section 2. All mimics' dynamics corresponds to the dynamic produced by the numerical traditional simulation; therefore the movement's law in the mimics is based on the mathematical model.

To the objective of showing mimics in an user friendly environment, an interface is created where mimics can be reproduced in a compact fashion with no need of calculation or code running each time it is used. It is a simple interface with four buttons permitting to choose which mimic is reproduced. One advantage of this environment is that it can be saved in a CDF format compatible with Wolfram CDF Player®, free-distributed and light software that permits to reproduce Mathematica® files. Anyway, another file is created containing all models, simulations and mimic creation codes for all systems studied, where parameters and input references can be modified to visualize different types of response, allowing also the possibility to add different models for the same elements and/or models for different elements.

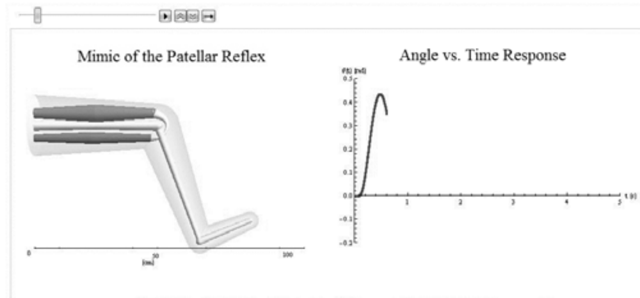

Fig. 20. Mimic of the patellar reflex response

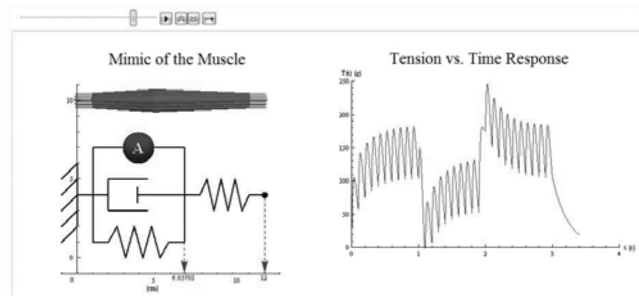

Fig. 21. Mimic of the muscle.

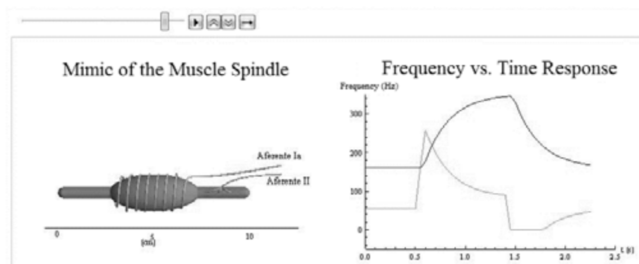

Fig. 22. Mimic of the muscle spindle.

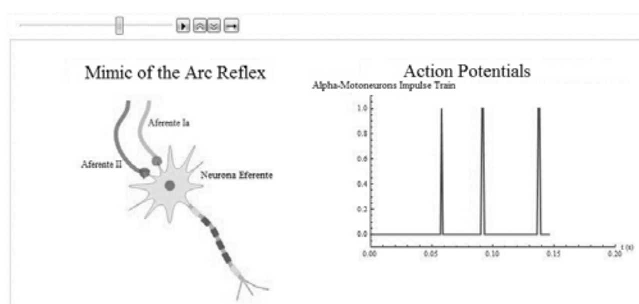

Fig. 23. Mimic of the arc reflex mechanism.

5 Conclusions

One of the primary objectives of this research is the application of engineering techniques such as modeling and simulation to biological and physiological systems like arc reflex. The accomplishment of this objective permitted the understanding of these systems' behavior from the engineering perspective.

Continuous and discrete-time models were obtained and analyzed for muscle contraction as actuator, muscle spindle as sensor, and arc reflex mechanism as controller for a general stretch reflex system and later particularized for the patellar reflex response. This stretch reflex model can be also expanded to other reflexes and functions, adding models of different receptors and effectors.

Modeling these systems permits the comprehension of their functioning, the analysis of their behavior and their possible application into other areas; for instance, building robots and exoskeletons that behaves as these biological systems which by nature are optimal or creating support systems for motor disabilities applying control theory on them, besides further applications in engineering and medicine fields.

One of the main contributions for these models using mimics is that all mimics' dynamics corresponds to the dynamic produced by the numerical traditional simulation; therefore the movement's law in the mimics is based on the mathematical model.

The computational mimics developed, represent a starting point for the creation of a simulator with a larger quantity of biological systems that permits its application with teaching purposes and health areas training.

Future work could be focused in the analysis of these dynamic models under some anomalies (simulated as perturbations) such as fatigues in the muscle, weaknesses in the muscle, and so on. This could enlarge the application of control theory on the presented models.

References

Aguirre-Gil I, Gómez C, Dugarte C, 2013, Sistema ascensor de cuatro pisos controlado mediante redes de Petri con fines didácticos, Revista Ciencia e Ingeniería, Vol. 34, No. 2, pp. 81-90.

Aguirre I, Amaro M, 2012, Sistema de movimiento del cursor del computador para personas con discapacidad, Revista Ciencia e Ingeniería, Vol. 33, No. 2, pp. 79-84.

Belandria M, Gómez C, Márquez R, Ríos M, 2012, Modelo promedio de un proceso de lodos activados en alternancia de fases para la remoción de nitrógeno en aguas residuales, Revista Ciencia e Ingeniería, Vol. 33, No. 2, pp. 105-112.

Beltrami E, 1997, Mathematics for dynamic modeling, Academic Press, San Diego, CA, USA, Second edition.

Ermentrout G, Terman D, 2010, Mathematical Foundations of Neuroscience, Springer Verlag, New York.

Hill A. V, 1938, The heat of shortening and the dynamic constants of muscle, Proceedings of the Royal Society London Biological Sciences, Vol. 126, No. 843, pp. 136–195.

Hodgkin A. L, Huxley A. F, 1952, A quantitative description of membrane current and its application to conduction and excitation in nerve, Vol.117, pp. 500–544.

Isermann R, 1989, Digital control systems, Springer Verlag, Berlin, 2nd edition.

Milhorn H. T, 1966, The application of control theory to physiological systems, W. B. Saunders Company, Philadelphia, PA.

Ogata K, 1998, Ingeniería de control moderna, Prentice Hall, Edo. de México, México, 3rd edition.

Ogata K, 2004, System dynamics, Prentice Hall, New Jersey, 4th edition.

Perreault E. J, Crago P. E, Kirsch R. F, 2000, Estimation of intrinsic and reflex contributions to muscle dynamics: a modeling study, IEEE Transactions on Biomedical Engineering, Vol. 47, pp. 1413–1421.

Ramírez V, Patete R, Barrera J, Venegas M, Dugarte G, 2011, Un modelo de simulación de una estación de gasolina, Revista Ciencia e Ingeniería, Edición Especial: Jornada de Modelado y Simulación, pp. 43-54.

Rodríguez-Millán J, Patete A, 2003a, 3D- simulations in linear and nonlinear dynamical control systems through computer animated mimics, Proc. of Cast and Complexity in Biological, Physical and Engineering Systems, 9th International Conference on Computer Aided Systems Theory EUROCAST 2003, pages 192–195.

Rodríguez-Millán J, Patete A, 2003b, Visualización tridimensional de sistemas de control mediante mímicos en Mathematica®, IV Congreso de Control y Automatización CAC'2003.

Rodríguez-Millán J, Garcia R, Patete A, Peña R, Prato F, Sánchez L, 2002, An integrated computational tool for nonlinear control, Symposium in Honor of Bruno Buchberger's 60th Birthday.

Rodríguez-Millán J, González C, Patete A, 2005a, Analysis, synthesis and mimetization of nonlinear control systems using Mathematica®, Cast and Tools for Robotics, Vehicular and Communication Systems, 10th International Conference on Computer Aided Systems Theory EUROCAST 2005, Vol. 20, pp. 316–320.

Rodríguez-Millán J, González C, Patete A, 2005b, Improved non-standard discretization methods for nonlinear dynamical control systems. Computer Aided Systems Theory EUROCAST 2005, Vol. LNCS 3643, pp. 608–613.

Santillán M, Páez-Hernández R, Delgado-Lezama R, 2003, A numeric study of the noise-induced tremor in a mathematical model of the stretch reflex, ScienceDirect Journal of Theoretical Biology, Vol. 222, pp. 99–115.

Shadmehr R, Wise S.P, 2005, Computational neurobiology of reaching and pointing, MIT Press, Cambrigde MA.

Recibido: *20 de septiembre de 2014*

Aceptado: *15 de enero de 2015*

Patete, Anna: *Ingeniero de Sistemas. Magister Scientiae en Control y Automatización en la Universidad de Los Andes, Mérida, Venezuela, en el 2002 y 2005, respectivamente. Doctor en Ingeniería, en el 2008, en TokyoDenkiUniversity, Japón. Actualmente es profesora ordinaria, nivel Agregado*

en el Departamento de Sistemas de Control, Facultad de Ingeniería, de la Universidad de Los Andes, Mérida, Venezuela. Sus principales áreas de investigación son: la teoría de control, sistemas de control a tiempo discreto, controladores adaptativos, y robótica.

Márquez, Pedro: *Ingeniero de Sistemas en la Universidad de Los Andes, Mérida, Venezuela, en el 2013. Actualmente realiza sus estudios de Master en Ingeniería Biomédica, enGhentUniversity, Bélgica. Sus principales áreas de investigación son: modelado de sistemas fisiológicos, ingeniería neural, interfaz cerebro-máquina. Correo electrónico: pedromarquezv@gmail.com.*

Rodríguez-Millán, Jesús: *Ingeniero en Electrónica en la Universidad Simón Bolívar, Venezuela. Estudió matemáticas en el Postgrado en Matemáticas de la Facultad de Ciencias de la Universidad Central de Venezuela. En 1989 se trasladó al Departamento de Sistemas de Control, Facultad de Ingeniería, Universidad de Los Andes, Mérida, Venezuela. Realizó sus estudios de doctorado en la Universidad Técnica de Budapest, Hungría, en 1999. Actualmente es profesor jubilado, nivel titular del Departamento de Sistemas Control de la Universidad de Los Andes. Sus principales áreas de investigación son: la teoría de sistemas dinámicos, sistemas no lineales, sistemas de control, computación simbólica y sistemas de control a tiempo discreto.Correo electrónico: jrmillan@ula.ve*