Artículo de Investigación. Revista Ciencia e Ingeniería. Vol. 38, No. 1 pp 41-52, diciembre - marzo, 2017.
ISSN 1316-7081. ISSN Elect. 2244-8780 Universidad de los Andes (ULA)

Nonlinear Implicit Self-Tuning Control applied to a Slider-Crank Mechanism

Control No Lineal Auto-Ajustable Implícito aplicado al Mecanismo Corredera-Biela-Manivela

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Abstract

In this research work, a new nonlinear mathematical model is proposed to represent the nonlinear dynamics of the slider-crank mechanism. Then, the nonlinear model is rewritten into a particular nonlinear class of mathematical model structure in the discrete-time case, defined by polynomial terms, with the purpose to control the mechanism in a large range of positional angles. Generalized minimum variance nonlinear control for the known system case, and implicit self-tuning nonlinear control based on generalized minimum variance for the presence of uncertainty parameters are used to regulate to a desired position the proposed nonlinear mathematical model for the slider-crank mechanism. This paper presents the first simulation results of the nonlinear implicit self-tuning control based on generalized minimum variance applied to a nonlinear mathematical model of a real system. The simulation results show the good performance of the system output response and the control law.

Key words: Generalized minimum variance, nonlinear systems, self-tuning control, slider-crank mechanism.

Resumen

En este trabajo de investigación se propone un nuevo modelo no lineal que permita representar las dinámicas no lineales intrínsecas del mecanismo corredera-biela-manivela. Este modelo no lineal se reescribe para satisfacer una estructura predefinida descrita por funciones polinómicas a tiempo discreto, con el propósito de luego controlar el mecanismo en un mayor rango de posiciones angulares. En el caso ideal donde los parámetros del sistema son todos conocidos, se muestra el uso del control no lineal por mínima varianza generalizada para regular el sistema. Por otra parte, para el caso donde los parámetros del sistema presentan incertidumbre, se propone el control no lineal auto-ajustable basado en mínima varianza generalizada para controlar el modelo matemático no lineal propuesto del mecanismo a una posición angular deseada. Este artículo presenta los primeros resultados, a través de simulaciones, del control no lineal auto-ajustable implícito basado en mínima varianza generalizada aplicado a un modelo matemático no lineal de un sistema real. Los resultados simulados demuestran el buen comportamiento de las dinámicas tanto de la salida del sistema como de la ley de control.

Palabras claves: Mínima varianza generalizada**,** sistemas no lineales**,** controlador auto-ajustable, mecanismo corredera-biela-manivela.

1 Introducción

Real world systems are mostly nonlinear systems and it is of interest to study the case of controlling nonlinear model by nonlinear control, and to analyze if the benefits are worthwhile than to use linear model and linear control, as the work done by (Jerez y col., 2005).

The slider-crank mechanism has a very wide usage in machine design. Some of the applications are found in internal combustion engines, in electrical switch gears, packaging and textile engineering (Yalcin 2014). Several control techniques have been presented in this system in the literature; however the control law is designed based on a linear model in the system.

The stability of implicit self-tuning control has been proved, for the linear discrete-time case, by the use of a Lyapunov function in (Patete y col., 2008a), where simulation results controlling linear model under parameter uncertainties showed good performance. Saitoh and Furuta (Saitoh y col., 2007, Furuta y col., 2009) implemented the generalized minimum variance combined with self-tuning control algorithm to a real slider-crank mechanism, using a linear model for the real system. The linear model was obtained by the projection method approach (Blajer 1996), and the results show good performance for control objectives near to initial conditions. Letter on, the control algorithm results were extended to the MIMO (multiple input multiple output) linear case in (Sugiki y col., 2008, Furuta y col., 2011, Ohata y col., 2014), applying the implicit self-tuning control to an engine model.

Bilinear systems are the simplest class of nonlinear systems and can also be regarded as a practical starting point for the study of other nonlinear systems. A new algorithm was proposed, based on the results in (Patete y col., 2008a), for the self-tuning control combining recursive parameter estimation and generalized minimum variance criterion, for a class of bilinear systems in (Patete y col., 2008b, 2011), and also for an extended and more relaxed class of bilinear systems, where the control action could be presented only in the bilinear term in (Patete y col., 2010, 2014). The validity and performance of the algorithm were demonstrated through simulation results considering as a case of study the mathematical bilinear model for nuclear fission system (Patete y col., 2014).

A nonlinear class of systems was defined, and a nonlinear implicit self-tuning control of the defined class was presented in (Patete y col., 2015), yet only theoretical results were given.

This paper presents the first simulation results of the nonlinear implicit self-tuning control based on generalized minimum variance applied to a nonlinear mathematical model of a real system, the slider-crank mechanism, based on the nonlinear control algorithm proposed in (Patete y col., 2015).

The paper is organized as follows: section 2 presents the basic theory of the nonlinear implicit self-tuning control based on a generalized minimum variance algorithm for a class of nonlinear mathematical models. In section 3, a new nonlinear mathematical model for the slider-crank mechanism is proposed. As a case of study, the nonlinear implicit self-tuning control based on generalized minimum variance is applied to the nonlinear mathematical model for the slider-crank mechanism in section 4, and simulation results are showed. Some remarks are given at the end.

Nonlinear Implicit Self-Tuning Control based on the Generalized Minimum Variance for a Class of Nonlinear Mathematical Models

A nonlinear implicit self-tuning controller has been proposed (Patete y col., 2015) to regulate a defined class of nonlinear system models. The proposed algorithm is presented in this section as follows:

Consider the general, Single Input Single Output (SISO), structured in the discrete-time case of a nonlinear system model as in (1),

$$A(z,q)y_k = B(z,q)u_k\,, \tag{1}$$

where y_k is the output signal of the process, u_k is the input signal, z denotes the time shift operator: $z^{-d}y_k = y_{k-d}$; $A(z,q)$ and $B(z,q)$ are polynomials of the form:

$$A(z,q) = 1 + \sum_{i=1}^{n} a_i(q) z^{-i},$$

$$B(z,q) = \sum_{i=0}^{m} b_i(q) z^{-i},$$

n is the order of polynomial $A(z,q)$ and m is the order of polynomial $B(z,q)$, q in the general case is a function of the input and output signal of the process as in (2),

$$q = h(y_k, u_k)\,. \tag{2}$$

where $h(y_k, u_k)$ is any function (linear or nonlinear). Then, to obtain the nonlinear model to fit in the defined class (Patete y col., 2015), the function $h(y_k, u_k)$ is restricted to be a function depending only of the output process data, i.e. $h(y_k)$, then q is as follows:

$$q = h(y_k) = \sum_{i=1}^{n} h_{1i} y_{k+n-1}^{i-1}\,, \tag{6}$$

where h_{1i} are the coefficients contained in the functions: $a_i(q)$ and $b_i(q)$, of system model (1).

In general, the class of nonlinear systems is defined as a SISO time invariant model (7) with the following structure (Patete y col., 2015):

$$y_{k+n} + \sum_{i=1}^{n} a_i(q) y_{k+n-i} = b(q) u_k\,, \tag{7}$$

with q as in (6).

The generalized minimum variance control based on the concept of discrete-time sliding mode is proposed in (Patete y col., 2015) for the defined class of nonlinear systems (7) as:

Consider the general nonlinear model (7), if:

$$
\begin{aligned}
&a_1(q)=\sum_{i=1}^{n} a_{1i} y_{k+n-1}^{i-1},\\
&a_2(q)=\sum_{i=1}^{n} a_{2i} y_{k+n-2}^{i-1},\\
&\vdots\\
&a_n(q)=\sum_{i=1}^{n} a_{ni} y_{k}^{i-1},\\
&b\,(q)=\sum_{i=0}^{n} b_i y_{k}^{i},
\end{aligned} \tag{8}
$$

then, substituting (8) in (7), the following (9) is obtained

$$
\begin{aligned}
&A(z^{-1})y_k+\sum_{i=2}^{n}\Delta_{y_k^i}(z^{-1})y_k^i=\\
&z^{-d}\left(B(z^{-1})u_k+\sum_{i=1}^{n}\Gamma_{y_k^i u_k}(z^{-1})y_k^i u_k\right),
\end{aligned} \tag{9}
$$

where

$$
\begin{aligned}
&A(z^{-1})=1+\sum_{i=1}^{n} a_{i1} z^{-i},\\
&\Delta_{y_k^2}(z^{-1})=\sum_{i=1}^{n} a_{i2} z^{-i},\\
&\Delta_{y_k^3}(z^{-1})=\sum_{i=1}^{n} a_{i3} z^{-i},\\
&\vdots\\
&\Delta_{y_k^n}(z^{-1})=\sum_{i=1}^{n} a_{in} z^{-i},\\
&B(z^{-1})=b_0,\\
&\Gamma_{y_k u_k}(z^{-1})=b_1,\\
&\Gamma_{y_k^2 u_k}(z^{-1})=b_2,\\
&\vdots\\
&\Gamma_{y_k^n u_k}(z^{-1})=b_n.
\end{aligned}
$$

The control objective of the control law is to minimize the variance of the linear controlled sliding mode variable s_{k+d}, defined as (10):

$$s_{k+d}=C(z^{-1})(y_{k+d}-r_{k+d})+Q(z^{-1})u_k, \tag{10}$$

where polynomials $C(z^{-1})$ and $Q(z^{-1})$ are defined as in (11) and (12) respectively,

$$C(z^{-1})=1+\sum_{i-1}^{n} c_i z^{-i}, \tag{11}$$

$$Q(z^{-1})=q_0(1-z^{-1}), \tag{12}$$

Polynomials $C(z^{-1})$ and $Q(z^{-1})$ are designed, so that the error signal e_k, defined as (13), vanish:

$$e_k=y_k-r_k, \tag{13}$$

where r_k is the reference signal.

Using the Diophantine equation (Chang y col., 1968):

$$C(z^{-1})=A(z^{-1})E(z^{-1})+z^{-d}F(z^{-1}), \tag{14}$$

where,

$$
\begin{aligned}
&F(z^{-1})=\sum_{i=1}^{n} f_{\,i-1} z^{-n+1},\\
&E(z^{-1})=\sum_{i=1}^{d} e_{\,d-1} z^{-d+1}.
\end{aligned}
$$

equation (14) is rewritten as (15):

$$
\begin{aligned}
&C(z^{-1})y_{k+d}=F(z^{-1})y_k+\sum_{i=2}^{n}P_{y_k^i}(z^{-1})y_{k+d}^i+\\
&\quad E(z^{-1})B(z^{-1})u_k+\sum_{i=1}^{n}P_{y_k^i u_k}(z^{-1})y_k^i u_k,
\end{aligned} \tag{15}
$$

where:

$$
\begin{aligned}
&P_{y_k^i}(z^{-1})=-E(z^{-1})\Delta_{y_k^i}(z^{-1}),\quad i=2,3,\ldots,n\\
&P_{y_k^i u_k}(z^{-1})=E(z^{-1})\Gamma_{y_k^i u_k}(z^{-1}),\quad i=1,2,\ldots,n.
\end{aligned}
$$

Combining (15) and (9), the variable s_{k+d} is:

$$
\begin{aligned}
&s_{k+d}=F(z^{-1})y_k+\sum_{i=2}^{n}P_{y_k^i}(z^{-1})y_{k+d}^i+G(z^{-1})u_k\\
&\quad+\sum_{i=1}^{n}P_{y_k^i u_k}(z^{-1})y_k^i u_k-C(z^{-1})r_{k+d},
\end{aligned} \tag{16}
$$

where $G(z^{-1})=E(z^{-1})B(z^{-1})+Q(z^{-1})$.

Then, the generalized minimum variance control input required to vanish s_{k+d} in (10) is given by (Patete y col., 2015),

$$u_k = -\frac{F(z^{-1})y_k + \sum_{i=2}^{n} P_{y_k^i}(z^{-1})y_{k+d}^i - C(z^{-1})r_{k+d}}{G(z^{-1}) + \sum_{i=1}^{n} P_{y_k^i u_k}(z^{-1})y_k^i} \tag{17}$$

For the implicit self-tuning control, system (9) is considered as a system with the same structure; however uncertainty parametric is taken into consideration. Then, the parameters of the nominal control law (17) are estimated each sampled time.

The closed-loop stability of self-tuning control of the defined nonlinear systems class, based on the generalized minimum variance criterion, is given by the following recursive estimation equations (Patete y col., 2015):

$$\begin{aligned}\hat{\theta}_k &= \hat{\theta}_{k-1} + \\ &\frac{\Gamma_{k-1}\phi_{k-d}}{1+\phi_{k-d}^T\Gamma_{k-1}\phi_{k-d}}[s_k + C(z^{-1})r_k - \phi_{k-d}^T\hat{\theta}_{k-1}],\end{aligned} \tag{18}$$

$$\Gamma_k = \Gamma_{k-1} - \frac{\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^T\Gamma_{k-1}}{1+\phi_{k-d}^T\Gamma_{k-1}\phi_{k-d}}, \tag{19}$$

where

$$\begin{aligned}\phi_k^T = [&y_k,\ldots,y_{k-n+1},y_k^2,\ldots,y_{k-n-d+1}^2,\ldots,\\ &y_k^n,\ldots,y_{k-n-d+1}^n,u_k,\ldots,u_{k-d+1},y_ku_k,\ldots,\\ &y_{k-d+1}u_{k-d+1},y_k^2u_k,\ldots,y_{k-d+1}^2u_{k-d+1},\\ &y_k^nu_k,\ldots,y_{k-d+1}^nu_{k-d+1}]\end{aligned} \tag{20}$$

is the vector containing measured output and control signal data,

$$\begin{aligned}\theta^T = [&f_0,\ldots,f_{n-1},P_{y_k^2 0},\ldots,P_{y_k^2 n+d-1},\ldots,\\ &P_{y_k^n 0},\ldots,P_{y_k^n n+d-1},g_0,\ldots,g_{d-1},P_{y_ku_k 0},\ldots,\\ &P_{y_ku_k d-1},\ldots,P_{y_k^nu_k 0},\ldots,P_{y_k^nu_k d-1}]\end{aligned} \tag{21}$$

is the vector containing the controller parameters, and

$$\begin{aligned}\hat{\theta}^T = [&\hat{f}_0,\ldots,\hat{f}_{n-1},\hat{P}_{y_k^2 0},\ldots,\hat{P}_{y_k^2 n+d-1},\ldots,\\ &\hat{P}_{y_k^n 0},\ldots,\hat{P}_{y_k^n n+d-1},\hat{g}_0,\ldots,\hat{g}_{d-1},\hat{P}_{y_ku_k 0},\ldots,\\ &\hat{P}_{y_ku_k d-1},\ldots,\hat{P}_{y_k^nu_k 0},\ldots,\hat{P}_{y_k^nu_k d-1}]\end{aligned} \tag{22}$$

is the estimate of θ.

The controller uses identified parameters (Patete y col., 2015) as follows:

$$u_k = -\frac{\hat{F}(z^{-1})y_k + \sum_{i=2}^{n} \hat{P}_{y_k^i}(z^{-1})y_{k+d}^i - C(z^{-1})r_{k+d}}{\hat{G}(z^{-1}) + \sum_{i=1}^{n} \hat{P}_{y_k^i u_k}(z^{-1})y_k^i} \tag{23}$$

The given algorithm is based on the idea of the discrete-time sliding mode control concept (Furuta 1990, 1993).

3 A Nonlinear Mathematical Model for the Slider-Crank Mechanism

The slider-crank mechanism is a system that can convert linear forces into rotational torque. The slider-crank mechanism is used in many real systems like automobile engines. Fig. 1 shows a schematic model of the slider-crank mechanism. The slider is restricted by its direction of motion on the x-axis and the center of the wheel, for the rotation axis, is fixed at the origin. Table 1 shows the model variables and Table 2 the parameters of the slider-crank mechanism.

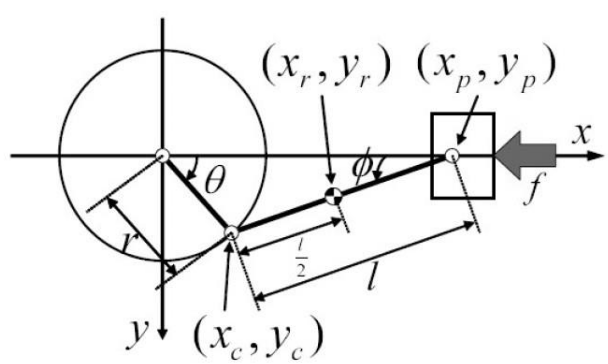

Fig. 1. Schematic model of the slider-crank mechanism

Table 1. Variables of slider-crank mechanism

(x_p, y_p)	**[m]**	**Slider position**
(x_r, y_r)	[m]	Connecting rod COG
θ	[rad]	Wheel angle
ϕ	[rad]	Connecting rod angle
f	[N]	Input force to slider

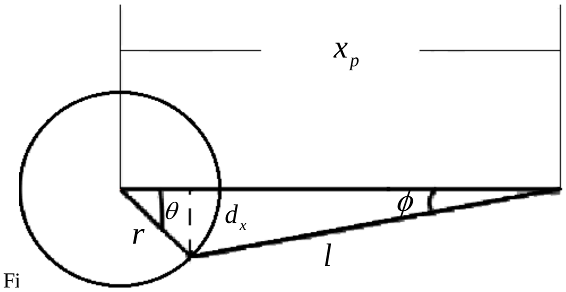

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Table 2. Parameters of slider-crank mechanism

m_p	**[Kg]**	**Slider mass**
d_p	[N.s/m]	Slider viscous coefficient
m_r	[Kg]	Connecting rod mass
J_r	[Kg.m^2]	Inertia around COG of connecting rod
l	[m]	Connecting rod length
m_c	[Kg]	Wheel mass
d_c	[N.m.s/rad]	Wheel viscous coefficient
J_c	[Kg.m^2]	Inertia around COG of wheel
r	[m]	Wheel radius

A mathematical model for the slider-crank mechanism was proposed by (Saitoh y col., 2007, Furuta y col., 2009) using the projection method (Blajer 1996), where after some consideration and conditions this nonlinear system is represented by a linear model. As it's known, a nonlinear system represented by a linear model may be controlled and assured its global stability only near its operating point and perhaps some nonlinear dynamics get lost. In this work a new nonlinear model is proposed to represent the nonlinear dynamics of the slider-crank mechanism.

The slider mass m_p and the connecting rod mass are assumed to be one hole mass with COG in m_p. Then, the force f applied to m_p is equal to the force f applied at the end of the connecting rod.

Based on Fig. 2, where x_p is the base and d_x the high of the formed triangle respectively; using Newton's second law for the translational movement, and knowing that slider viscous coefficient force d_p works against the input force f ,

$$\sum F = m a, \tag{24}$$

$$f - d_p \dot{x}_p = m_p \ddot{x}_p. \tag{25}$$

From the formed triangle the following relation (26) is obtained:

$$x_p = r\,Cos(\theta) + l\,Cos(\phi). \tag{26}$$

The first and second derived of (26) are,

$$\dot{x}_p = -r\,Sin(\theta)\dot{\theta} - l\,Sin(\phi)\dot{\phi}, \tag{27}$$

$$\ddot{x}_p = -r\left[Cos(\theta)\dot{\theta} + Sin(\theta)\ddot{\theta}\right] - l\left[Cos(\phi)\dot{\phi} + Sin(\phi)\ddot{\phi}\right]. \tag{28}$$

Substituting (27) and (28) in (26), the following (25) is obtained,

$$f - d_p\left[-r\,Sin(\theta)\dot{\theta} - l\,Sin(\phi)\dot{\phi}\right] = m_p\left[-r\left[Cos(\theta)\dot{\theta} + Sin(\theta)\ddot{\theta}\right] - l\left[Cos(\phi)\dot{\phi} + Sin(\phi)\ddot{\phi}\right]\right]. \tag{29}$$

As the variable to be controlled is the wheel angle θ, then (29) should depend only on θ. Using the triangle relation (30), (31) is obtained:

$$d_x = l\,Sin(\phi) = r\,Sin(\theta), \tag{30}$$

$$Sin(\phi) = \frac{r}{l} Sin(\theta). \tag{31}$$

To obtain a relation for $\cos(\phi)$, the cosines law is used from Fig. 2,

$$d_x^{\,2} = l^2 + l^2\,Cos^2(\phi) - 2l\left[l\,Cos(\phi)\right]Cos(\phi). \quad (32)$$

From (30),

2 $$d_x^{\,2} = r^2\,Sin^2(\theta),$$
(33)

substituting (33) in (32), (34) is obtained,

$$r^2\,Sin^2(\theta) = l^2 + l^2\,Cos^2(\phi) - 2l^2Cos^2(\phi). \quad (34)$$

Manipulating (34), (35) is calculated,

$$Cos(\phi) = \sqrt{1 - \frac{r^2}{l^2}Sin^2(\theta)}. \quad (35)$$

Using Newton's Binomial Theorem for the right side of (35)

$$\left[1 - \frac{r^2}{l^2}Sin^2(\theta)\right]^{\frac{1}{2}} = 1^{\frac{1}{2}} - \frac{1}{2}1^{\frac{1}{2}-1}\frac{r^2}{l^2}Sin^2(\theta) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}1^{\frac{1}{2}-2}\frac{r^4}{l^4}Sin^4(\theta) + \ldots \quad (36)$$

The superior terms of (36) are insignificant numerically; therefore (36) may be rewritten in the following way:

$$\left[1 - \frac{r^2}{l^2}Sin^2(\theta)\right]^{\frac{1}{2}} = 1^{\frac{1}{2}} - \frac{1}{2}\frac{r^2}{l^2}Sin^2(\theta), \quad (37)$$

then,

$$Cos(\phi) = \sqrt{1 - \frac{r^2}{l^2}Sin^2(\theta)} = 1^{\frac{1}{2}} - \frac{1}{2}\frac{r^2}{l^2}Sin^2(\theta). \quad (38)$$

From (31), using (38), (40) is computed

$$\dot{\phi} = \frac{r\,Cos(\theta)}{l\,Cos(\phi)}\dot{\theta}, \quad \mathbf{(39)}$$

$$\dot{\phi} = \frac{2\,l\,r\,Cos(\theta)}{2\,l^2 - r^2Sin^2(\theta)}\dot{\theta}, \quad \mathbf{(40)}$$

Deriving (39),

$$\ddot{\phi} = \frac{\frac{r}{l}\left(Cos(\theta)\ddot{\theta} - Sin(\theta)\dot{\theta}\right) + Sin(\phi)\dot{\phi}}{Cos(\phi)}, \quad \mathbf{(41)}$$

substituting (31), (38) and (40) in (41), (42) is obtained,

$$\ddot{\phi} = \frac{2\,l\,r\left(2\,l^2\ddot{\theta}\,Cot(\theta) - 2\,l^2\dot{\theta}\right)}{4l^4Csc(\theta) - 4l^2r^2Sin(\theta) + r^4Sin^3(\theta)} + \frac{2\,l\,r\left(r^2Sin^2(\theta)\dot{\theta}\right)}{4l^4Csc(\theta) - 4l^2r^2Sin(\theta) + r^4Sin^3(\theta)} - \frac{2\,l\,r\left[r\,Cos(\theta)\left(rSin(\theta)\ddot{\theta} - 2\,l\dot{\theta}\right)\right]}{4l^4Csc(\theta) - 4l^2r^2Sin(\theta) + r^4Sin^3(\theta)} \quad \mathbf{(42)}$$

Finally, to obtain the translational movement equation, (31), (38), (40) and (42) are substituted in (29):

$$\ddot{\theta}=\frac{f+2l\,m_p\dot{\theta}+d_p r\,Sin(\theta)\dot{\theta}}{m_p r\left(-Sin(\theta)-\frac{2l\,rSin(2\theta)}{4l^2-r^2+r^2Cos(2\theta)}\right)}+\frac{2m_p rCos(\theta)\dot{\theta}}{m_p r\left(-Sin(\theta)-\frac{2l\,rSin(2\theta)}{4l^2-r^2+r^2Cos(2\theta)}\right)}+\frac{\frac{16m_p l^2r^3Sin^2(\theta)Cos(\theta)\dot{\theta}}{\left(4l^2-r^2+r^2Cos^2(2\theta)\right)^2}}{m_p r\left(-Sin(\theta)-\frac{2l\,rSin(2\theta)}{4l^2-r^2+r^2Cos(2\theta)}\right)}+\frac{\frac{2l\dot{\theta}\left(d_p r^2\,Sin(2\theta)-4l^2m_p\right)}{4l^2-r^2+r^2Cos(2\theta)}}{m_p r\left(-Sin(\theta)-\frac{2l\,rSin(2\theta)}{4l^2-r^2+r^2Cos(2\theta)}\right)}. \quad (43)$$

For the rotational movement:

$$\sum\tau=J\ddot{\theta}, \quad (44)$$

$$\tau_f-\tau_{dc}=J\ddot{\theta}, \quad (45)$$

$$f\;Sin(\theta)-d_c\dot{\theta}=\frac{2}{5}m_c\,r^2\ddot{\theta}, \quad (46)$$

$$\ddot{\theta}=\frac{5\left(d_c\dot{\theta}-r\;f\;Sin(\theta)\right)}{2m_c\,r^2}. \quad (47)$$

Equaling (43) and (47), the first order no linear model for the slider-crank mechanism is obtained and presented in (48),

$$\dot{\theta}=\frac{\begin{gathered}\left[1000\,f^3l^2m_pSin[\theta]Tan[\theta]\right.\\ \left(2l^2-r^2+r^2Cos[2\theta]\right)\\ \left(4l^2-r^2+r^2Cos[2\theta]\right)^2\\ \left(Cos[\theta]+Cos[3\theta]\right)\\ \left.\left(\left(4f\,l^3-f\,l\,r^2\right)Cos[\theta]+f\,r^3Sin^4[\theta]\right)\right]\end{gathered}}{\begin{gathered}\left[-r^5m_c\left(16l^2-5r^2\right)\left(24l^2-5r^2\right)\left(4l^2-r^2\right)\right.\\ \left(-8l^2r+3lr^3+\left(32l^4-4l^2r^2+r^4\right)Cos[\theta]\right)\\ \left(8lCos[2\theta]\left(2l^2-r^2\right)+rCos[3\theta]\left(8l^2-3r^2\right)\right)\\ \left(1+2Cos[2\theta]\right)\left(2r^2d_pm_c-5d_cm_p\right)^4\\ \left[r+8lCos[\theta]+2rCos[2\theta]+\right.\\ \left.8lCos[3\theta]+2rCos[4\theta]\right]\\ \left[32l^4r^2d_pm_cSin[\theta]+m_p\left(-80l^4Sin^4[\theta]d_c\right.\right.\\ \left.\left.\left.+r^5\left(2lCos[4\theta]+rCos[5\theta]\right)m_c\right)\right]\right]\end{gathered}} \quad (48)$$

The obtained nonlinear model (48) is a first order dynamical model, which is simpler than other models presented in the literature derived from the Euler-Lagrange technique, e.g. (Yalcin 2014). However, this model (48) captures the natural nonlinear behavior of the real system showed in Fig. 1.

Cases of Study: Slider-Crank Mechanism

In this section, simulation results are given for the application of the proposed nonlinear control: i) generalized minimum variance nonlinear control input (17), and ii) the set of equations (18)-(20) for the nonlinear implicit self-tuning, to the slider-crank mechanism nonlinear model proposed in this work.

First, the slider-crank mechanism model given in (48) should be transformed to a model represented polynomial terms as in (9). For that purpose the nonlinear terms in (48) are substituted by the respective Taylor Series. In this case only the first two terms of each series are considered, e.g.

$$Sin(\theta)=\theta+\frac{\theta^3}{6}.$$

Then, to represent the model in the discrete-time case as in (9), with T_0 representing the sampling-time period:

$$\theta(t)=\theta(kT_0),$$

and using Euler approximation for the first derivate:

$$\dot{\theta}(t)\approx\frac{\theta((k+1)T_0)-\theta(kT_0)}{T_0},$$

the following discrete-time model (49) is obtained,

$$\begin{aligned}&A(z^{-1})\theta_{kT_0}+\Lambda_{\theta^2}(z^{-1})\theta_{kT_0}^2+\Lambda_{\theta^3}(z^{-1})\theta_{kT_0}^3+\\&\Lambda_{\theta^4}(z^{-1})\theta_{kT_0}^4=z^{-d}\Big[B(z^{-1})f_{kT_0}+\\&\Upsilon_{\theta^2 f}(z^{-1})\theta_{kT_0}^2 f_{kT_0}+\Upsilon_{\theta^4 f}(z^{-1})\theta_{kT_0}^4 f_{kT_0}+\\&\Upsilon_{\theta^6 f}(z^{-1})\theta_{kT_0}^6 f_{kT_0}+\Upsilon_{\theta^8 f}(z^{-1})\theta_{kT_0}^8 f_{kT_0}+\\&\Upsilon_{\theta^{10} f}(z^{-1})\theta_{kT_0}^{10} f_{kT_0}+\Upsilon_{\theta^{12} f}(z^{-1})\theta_{kT_0}^{12} f_{kT_0}+\\&\Upsilon_{\theta^{14} f}(z^{-1})\theta_{kT_0}^{14} f_{kT_0}\Big],\end{aligned}\tag{49}$$

where,

$$A(z^{-1})=1-z^{-1},$$

$$\Lambda_{\theta^2}(z^{-1})=$$
$$\frac{2d_p m_c l r^2(l+r)-5d_c m_p(1+lr)}{4l^2r^2m_c m_p}z^{-1}-$$
$$\frac{2d_p m_c l r^2(l+r)-5d_c m_p(1+lr)}{4l^2r^2m_c m_p}z^{-2},$$

$$\Lambda_{\theta^3}(z^{-1})=$$
$$-\frac{l^2+lr+r^2}{2l^2}z^{-1}-\frac{l^2+lr+r^2}{2l^2}z^{-2},$$

$$\Lambda_{\theta^4}(z^{-1})=$$
$$\frac{5d_c m_p(l^2+4lr+6r^2)-2l^2r^2d_p m_c}{24l^2r^2m_c m_p}z^{-1}$$
$$-\frac{r\left(2d_p m_c(4l^2+6lr+3r^2)-15d_c m_p\right)}{24l^3m_c m_p}z^{-1},$$

$$B(z^{-1})=-\frac{T_0}{2rm_p},$$

$$\Upsilon_{\theta^2 f}(z^{-1})=\frac{T_o\left(2r^2m_c-5lm_p(l+r)\right)}{4l^2rm_c m_p},$$

$$\Upsilon_{\theta^4 f}(z^{-1})=$$
$$\frac{T_o\left(-3r^4m_c+5lm_p(2l^3+5l^2r+6lr^2+3r^3)\right)}{24l^4rm_c m_p},$$

$$\Upsilon_{\theta^6 f}(z^{-1})=$$
$$-\frac{5T_o(9+36l^3r+162l^2r^2+10lr^3+81r^4)}{1296l^4rm_c},$$

$$\Upsilon_{\theta^8 f}(z^{-1})=\frac{5T_o r(7l^2+4lr+12r^2)}{288l^4m_c},$$

$$\Upsilon_{\theta^{10} f}(z^{-1})=-\frac{5T_o r(2l^2+9r^2)}{864l^4m_c},$$

$$\Upsilon_{\theta^{12} f}(z^{-1})=-\frac{5T_o(l^3+12r^3)}{10368l^4m_c},$$

$$\Upsilon_{\theta^{14} f}(z^{-1})=-\frac{5T_o r^3}{20736l^4m_c},$$

$$d=1.$$

Then from (17), the generalized minimum variance control, to this particular model (49), is (50)

$$u_k = -\frac{F(z^{-1})\theta_k + P_{\theta^2}(z^{-1})\theta_{k+d}^2 + ... + P_{\theta^4}(z^{-1})\theta_{k+d}^4 - C(z^{-1})r_{k+d}}{G(z^{-1}) + P_{\theta^2 f}(z^{-1})\theta_k^2 + \ldots + P_{\theta^{14} f}(z^{-1})\theta_k^{14}}, \quad (50)$$

where,

$$P_{\theta^i}(z^{-1}) = -\Lambda_{\theta^i}\left(z^{-1}\right)E\left(z^{-1}\right) \quad i = 2,3,4.$$

$$P_{\theta^j f}(z^{-1}) = \Upsilon_{\theta^j f}\left(z^{-1}\right)E\left(z^{-1}\right) \quad j = 2,4,\ldots,14.$$

$$G(z^{-1}) = B\left(z^{-1}\right)E\left(z^{-1}\right) + Q\left(z^{-1}\right).$$

with j pair.

In this mechanism some parameters may be measured and some others not; as are the cases for the viscous coefficients, where only an interval of values for that parameters are known. The parameter values in this case of study are show in Table 3.

Table 3. Parameter values for the slider-crank mechanism model

m_p	**[Kg]**	**0.3**
d_p	[N.s/m]	[0.005; 0.07]
l	[m]	0.35
m_c	[Kg]	2.852
d_c	[N.m.s/rad]	[0.001; 0.05]
r	[m]	0.11

The sampling-time period is assumed as $T_0 = 0.0001$ s, the output angle is initiated in $\theta(0) = \frac{20\pi}{180}$. For the control design the following polynomials are chosen:

$$C(z^{-1}) = 1 + 0.5263z^{-1}, \quad (51)$$

$$Q(z^{-1}) = 0.1(1 - z^{-1}). \quad (52)$$

As there are uncertainties in the viscous coefficients, the implicit self-tuning control (18), (19) and (23) is used, with $\Gamma = \Gamma_0 = I$, and the desired output position (reference signal) is $\theta(\infty) = \frac{50\pi}{180}$. This final position is considered because it is far from the initial position, and linear controllers commonly don't stabilize the system in this case, as they show good performance only in cases near to the initial position (local control).

Fig. 3 shows the output responses of the slider-crank mechanism (49), when the generalized minimum variance (GMV) and the self-tuning (ST) algorithms are used, considering parametric uncertainties in the viscous coefficients. The control dynamics for both algorithms, generalized minimum variance control (GMVC) and self-tuning control (STC), are shown in Fig. 4. Finally, the sliding mode variable (SMV) for each case is presented in Fig. 5.

The generalized minimum variance control is not able to control the system to the reference because of the presence of uncertainty parameters, as it is shown in Fig. 3. On the contrary, the self-tuning control is able to reach the objective and the output dynamic presets good performance in steady-state. In Fig. 5 it is seems how the sliding mode variable vanishes (goes to zero) as the control objective is reached when the self-tuning control is applied.

It is worth to mention that in the case where there are no parameter uncertainties, the generalized minimum variance control is able to control the system to the reference with good performance in steady-state.

The self-tuning control based on generalized minimum variance is able to control the nonlinear system in a large range of positional angles, while commonly linear controllers' don't do it.

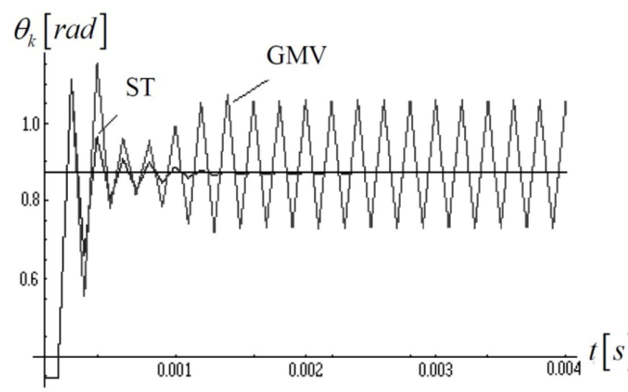

Fig. 3. Output dynamics of the slider-crank mechanism

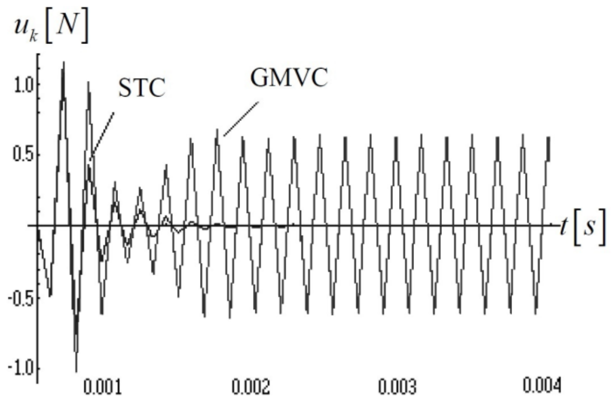

Fig. 4. Control dynamics

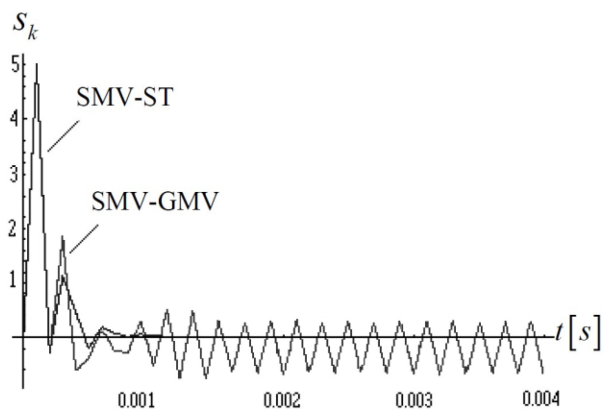

Fig. 5. Sliding mode variable dynamics

5 Conclusions

A new nonlinear mathematical model was proposed for the slider-crank mechanism. The nonlinear model was rewritten into the particular nonlinear class of mathematical model structure in the discrete-time case. Generalized minimum variance nonlinear control and implicit self-tuning nonlinear control based on generalized minimum variance for the presence of uncertainty parameters were used to regulate to a desired position the proposed nonlinear model. The simulation results showed that the nonlinear self-tuning control is able to reach the angular position objective and the output dynamic presets good performance in steady-state, in spite of model parametric uncertainties and angle initial condition is far from the angular desired position.

6 Acknowledgments

This research was supported by Grants-in-Aid for Scientific Research (B) (Grant Number 24360166) of JSPS, Japan.

The authors acknowledge the comments and suggestions given by Dr. Akihiko Sugiki of Nikki Denso Co., Ltd, Japan, and Akira Ohata of Toyota Motor Corporation, Japan.

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Recibido: *10 de marzo 2015*

Aceptado: *30 de julio 2015*

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