Revista GEOGRÁFICA VENEZOLANA
Volumen 62(1)2021 enero-junio
ISSNe 2244-8853 ISSNp 1012-1617
RECIBIDO: julio, 2019. **ACEPTADO**: abril, 2020. pp. 176-196
https://doi.org/10.53766/RGV/2021.62.01.06

Desarrollo de un sistema de proyección local en el Ecuador continental para la generación de cartografía a detalle

Elaboration of a local projection system in mainland Ecuador for the generation of detailed cartography

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Resumen

Varios países utilizan el sistema de proyección UTM por las ventajas que tiene en la representación cartográfica para escalas menores a 1:25.000, debido a que estas escalas absorben las deformaciones. En la actualidad, este sistema proyectivo es utilizado junto a las técnicas topográficas clásicas y la tecnología GNSS para generar cartografía a detalle, ocasionando incompatibilidad entre las medidas en el terreno respecto a sus correspondientes en el plano. Por ello se propone crear un Sistema de Proyección Cartográfica Local en el Ecuador continental para cartografía escala 1:500 y 1:1.000, mediante la generación de zonas que consideren el límite cantonal y la altura media de las zonas urbanas. Como resultado se obtuvo un sistema proyectivo conformado por siete zonas, el cual disminuyó el valor de la deformación y aumentó el área de cumplimiento a nivel continental respecto a los resultados obtenidos al utilizar el sistema de proyección UTM.

PALABRAS CLAVE: plano topográfico local (PTL); Local Transversa de Mercator (LTM); graficismo.

Abstract

Several countries use the UTM projection system due to the advantages it has in cartographic representation for scales smaller than 1:25,000, because these scales absorb deformations. Currently this projective system is used together with classical topographic techniques and GNSS technology to generate detailed cartography, causing incompatibility between measurements on the field with respect to their corresponding ones on the map. For this reason, it is proposed to create a Local Cartographic Projection System in continental Ecuador for 1:500 and 1:1,000 scale cartography by generating areas that consider the cantonal limit and the average height of urban areas. As a result, a projective system consisting of seven zones was obtained, which decreased the deformation value and increased the area of compliance at the continental level with respect to the results obtained when using the UTM projection system.

KEY WORDS: local topographic map (LTM); Local Transverse Mercator (LTM); graphism.

1. Introduction

The UTM or Gauss-Kruger projection are conformed coordinates that are obtained by holomorphic functions w (z) (w: = u + iv ∊ C) with respect to complex algebra and complex analysis. In fact, holomorphic functions directly comply with the d'Alembert-Euler equations (Cauchy-Riemann equations) of conformal mapping as described by Grafarend (1995), (Stein and Weiss, 1968; Wells, 1982; Engelsman, 1984; Engels and Grafarend, 1995). The UTM projection system is used by several countries to generate large scale maps (Hager *et al.*, 1989) due to the advantages it illustrates in terms of cartographic representations, among which stand out: the conservation of angles (conformity), the ease for locate points and the small distortions suffered by the surfaces and distances in the equatorial zones (Snyder, 1987) as long as the cartography produced is smaller than the 1: 25,000 scale (Idoeta, 2002). This occurs due to the fact that the deformations will be absorbed by the scale and resolution of the cartography, not allowing to appreciate them graphically due to graphism (Millán, 2006). In the case of producing cartography at a scale greater than 1: 25,000, the UTM projection next to the terrestrial relief, will influence the representation of the distances on the plane, since the magnitude measured on it will not coincide with the measurement on the ground, preventing to fulfill the technical requirements that precision projects demand (Idoeta, 2002).

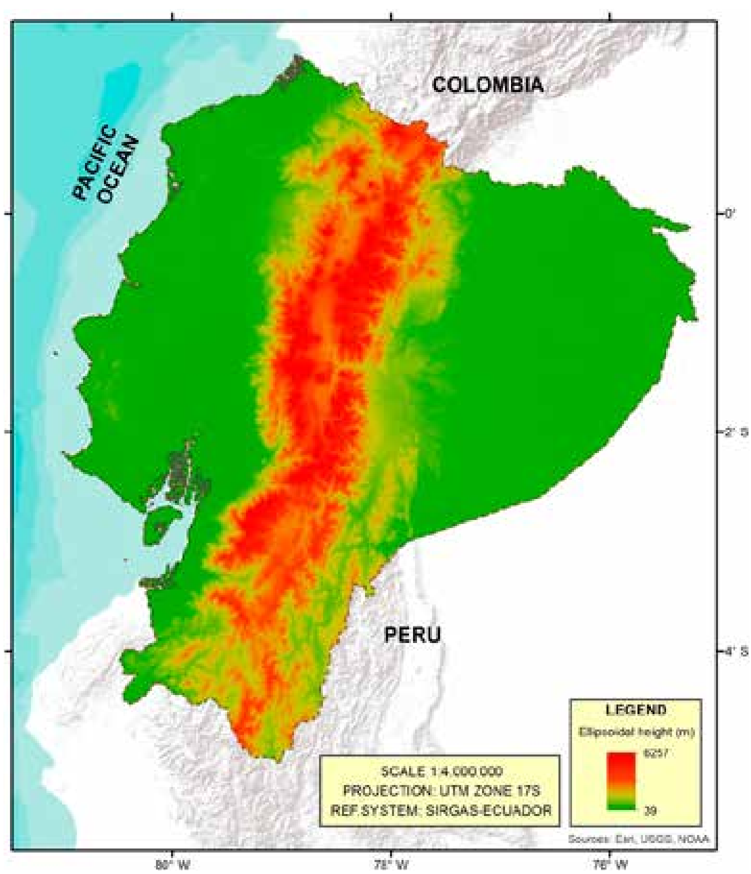
The cadastral plans are produced at scales that vary between 1: 100 and 1: 1,000 (Duarte, 2016), which is why the UTM projection may not be used to generate this type of cartography, since it will not be possible to comply with the technical requirements (Maling, 1992) as the deformations lack to be absorbed by the scale. Thus, they create the need to define a projection system that unifies the cartography in detail that occurs within the canton, since this administrative unit is responsible for generate and update the cadastral information (Duarte, 2016). The projective system will be of the compliant type due to the universality of the coordinates, the easy transformation in other similar systems that cover the same area, the need to have coordinates of local control points referred to a regional system for general mapping purposes and the planning of communication routes, among others (Blachut *et al.*, 1980).

The requirements of the detailed cartography together with the technological advance have caused that the topographic works combine the classic measurements with GNSS measurements in order to use the information gathered in a geographic information system (GIS) or to include it in the existing cartography, originating in the professionals the need to become familiar with a wide range of styles of positioning and data referenced to various datums and coordinate systems (Volker, 2009). The combination of techniques generates an inconsistency, since the GNSS measurements are vectors belonging to a World Geodetic Reference system, while the measurements obtained from the total stations are vectors represented in local coordinate systems. Between both types of vectors there is a relationship widely analyzed by geodesy and cartography that, when incorporated into a data processing methodology, will avoid inconsistencies in the inspection of cartography (Sánchez, 2008).

In order to combine the classic measurements of topography with GNSS measurements and to minimize the difference between the distance measured in the field with respect to their corresponding in the cartographic plane, the reductions are used to unify the measurements and refer them to the same reference surface. This is possible by using the deformation factors (Sánchez, 2008), which are dimensionless, as they lack to be easily interpreted and the deformation units such as m/km or PPM are used to express the relative error of a measurement, indicating the number of meters or millimeters of error respectively for each kilometer of survey. By applying the reductions in a practical way and to minimize the difference between the measured distances, the Local Transverse Mercator (LTM) projection associated with a Local Topographic Plan (LTP) is used, which modifies the factor in the central meridian using the methodology of the LTP that consists of calculating the reduction factor to the ellipsoid using the average height of the area where the projection will be applied (Cruz *et al.*, 2001).

The Republic of Ecuador is located on the northwestern side of South America and is traversed by the Andean Cordillera, which constitutes an impressive mountainous barrier with a width of 100 to 120 kilometers, exposing very steep external slopes of around 3,500 to 4,000 meters of elevation (Winckfll, 1982). The country presents the ideal conditions to apply the LTP methodology (FIGURE 1), which reduces the distances of the terrain to the ellipsoid chord, being this the most important correction of perform, of all the corrections of reduction of observables to the ellipsoid (Sánchez, 2008). By associating this methodology with an LTM projection where the height variation is controlled next to the projection field, it will be ensured that the deformations that occur comply with the planimetric precision and allow to generate cartography with quality in the detail.

FIGURE 1. Land relief of continental Ecuador



The National Cartography Law of Ecuador in article 18 of chapter II, does not specify the projection system that should be used to prepare special or thematic letters (Instituto Geográfico Militar, 1978). Furthermore, the regulations for the real estate cadastre of urban and rural appraisals and goods, in articles 10 and 11 of chapter III regarding the technical characteristics of cartographic maps, does not establish the projection system that should be used (Duarte, 2016). On the other hand, the municipalities have chosen to create geodesic networks for cadastral purposes, which has led that each one uses its own cartographic parameters. Such divergence hinders its use with common objectives, causing inconveniences when splicing the surveys in bordering areas of the cantons that have their own local projection, needing to perform complementary calculations of different gender to unify the projections and measurements made (Zakatov, 1997).

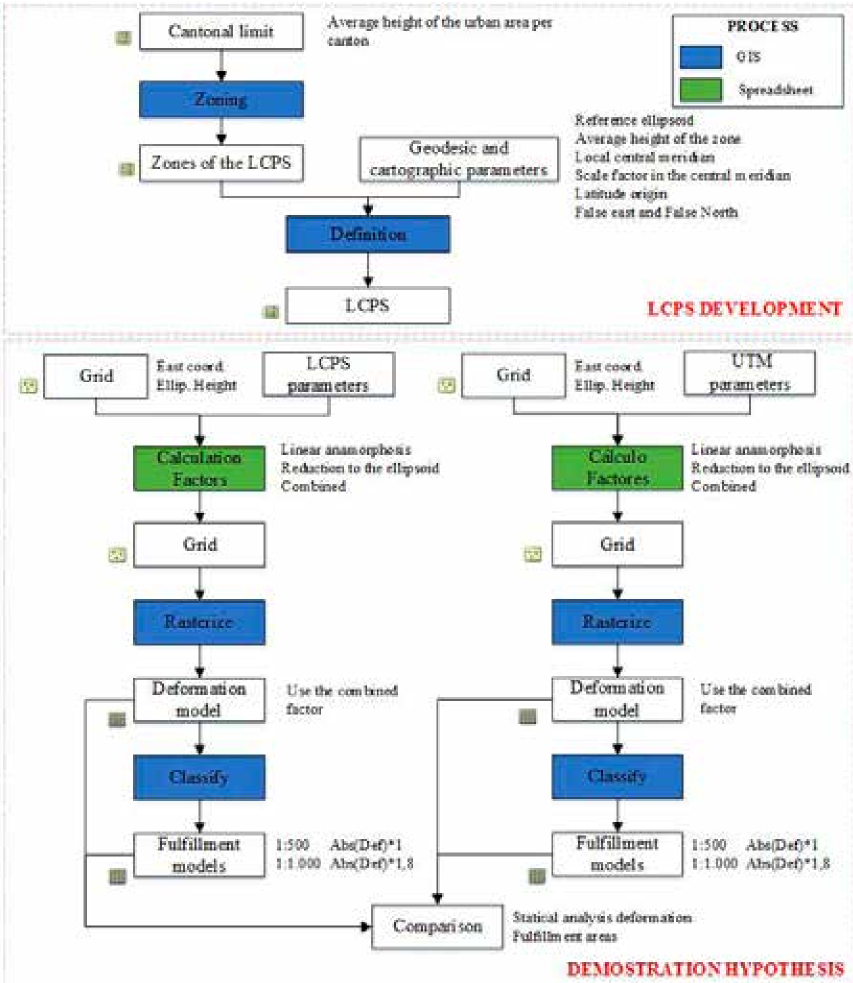
All these aforementioned reasons originate the need to create a Local Cartographic Projection System, in Continental Ecuador, by means of the mathematical analysis of the cartographic and geodetic parameters. With such defined local projections the planimetric precision that will be generated will be improved when using the system of UTM projection, unifying the detailed cartography that is produced within the municipalities and comply with the technical characteristics that the cadastral cartography requires.

2. Methodology

The generation of the Local Cartographic Projection System (LCPS) used the mathematical properties of the Transversal Mercator (TM) systems, due to the simplicity and facility for the calculation of the distortions and the easy use in large scale works to solve scientific problems of superior geodesy and first class triangulations (Zakatov, 1997; Luna *et al.*, 2017). This allowed also to maintain the familiarity of professionals who perform geographic information surveys with the characteristics of the UTM projection system. The considered methodology by Castillo (2015) where individualized areas based on the standardization of geodetic and cartographic parameters served as a guide for the present investigation with the difference that the objective of this projective system is to generate zones that guarantee compliance with the precision for cartography at scales 1: 500 and 1: 1,000. In this way the workflow has been obtained that is detailed in FIGURE 2.

In practice it has not been possible to define a projective system with a common origin that represents the points of the entire surface of the ellipsoid on a plane, as the distortions produced would be very large (Blachut *et al.*, 1980). Thus, this would result in the inevitable division of the surface into area, reason why which was zoned the LCPS considering three different variables. These have been first the cantonal limit, since it is the canton that has the competence to generate the cadastral cartography. The cadastral cartography is generated at scales of 1: 1,000 and 1: 500, depending on the density of buildings that exist in the city This variable was obtained in the shapefile format of the Institute of Statistics and Census (INEC). The urban area is the second variable, as in these areas the highest precision cartography is generated (1: 1,000 and 1: 500). This variable was obtained from the Military Geographical Institute (IGM) also in shapefile format. The third variable is the average height of each zone that is used to determine the scale factor due to the effect of the height for the LTM-LTP projections. This variable was obtained by transforming the orthometric heights of the digital elevation model Shuttle Radar Topography Mission (SRTM) at ellipsoidal heights, using the geoid ripple model EGM96 (Lemoine *et al.*, 1998).

FIGURE 2. Workflow of the methodology used in this study

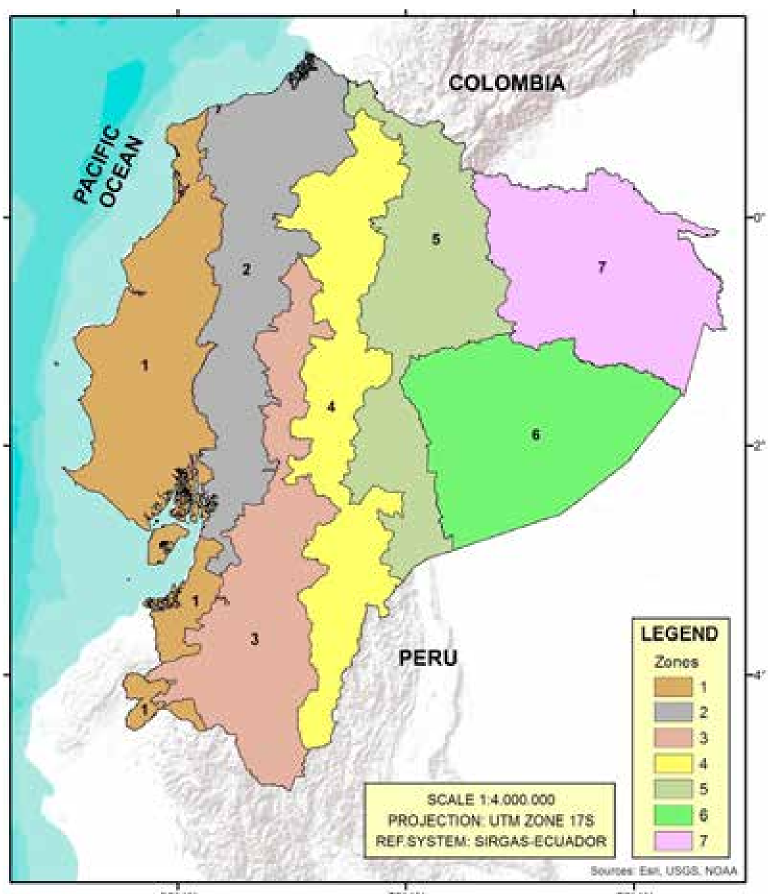


In order to determine the precision of the created model, GPS precision points were used, facilitated by the IGM, with which the Mean Square Error (MSE) was calculated, which is a measure of vertical accuracy (Maune *et al.*, 2007) of 7.05 meters. Such precision is adequate to define the average height of each zone as it will influence the scale factor due to the effect of the height in the sixth decimal figure, which means a millimeter variation in the deformations. The model of ellipsoidal heights covers 92.7% of the continental territory.

For the definition of the average height of each zone, the digital model of ellipsoidal heights was delimited using the polygon of the urban area, having several categories. The polygon that represents the cantonal head has been chosen as the first option. In the case of missing such category the next option has been the polygon of the parish head with the largest area. If there were the two aforementioned two polygons, the pointfill type of villages generated by the IGM has been used, where the point has been chosen which represented the cantonal head. For the digital elevation models obtained from each cut, the average of all the pixels was calculated. For the points, the value of the height extracted from the digital elevation model was used as the average height of the urban area. The value of the average height was added to the shapefile of cantons. Once obtaining all data, we proceeded to group the cantons that occupy the smallest width in an east - west direction and have a similar average height to reduce the distortions produced by the projection. In this way, the smallest number of possible areas was determined, with seven being the number of chosen zones (FIGURE 3).

The geodetic parameters used in the definition of the LCPS were the GRS80 ellipsoid, established by the Ecuadorian cartographic regulations, which is aligned with the SIRGAS-ECUADOR Reference System (Instituto Geográfico Militar, 2011). In order to obtain the average height of each zone, we calculated the average of the average heights of the urban area of all the cantons that belonged to the same area. The value of the average radius that was used to calculate the deformation factors, has been obtained by calculating the average of all the radios that have been by varying a degree within the territory of continental Ecuador plus the radii of the north and south ends (1.5 degrees

FIGURE 3. Areas of the Local Cartographic Projection System



north and 5.5 degrees south). Hereby, the obtained value has been of about 6'378.946 meters.

The cartographic parameters defined for the LCPS were the local central meridian at the level of the second, the scale factor in the central meridian of each zone that was calculated using equation 1 and it was expressed using seven decimal places. When this equation 1 is used the scale factor in the central meridian, it is called the scale factor due to height. The precision of these two parameters was based on the parameters used by the Municipal Ordinance of Quito (Concejo Metropolitano de Quito, 2007). The latitude origin of each zone was $0^0$00'00 "for the geographical location of continental Ecuador. The false east was 500,000 meters and the false north of 10'000,000 meters, based on Chilean regulations to define LTM-PTL projections (Cruz *et al.*, 2001). The cartographic and geodetic parameters of each zone that make up the LCPS are listed in TABLE 1.

$$K_{oh} = \frac{R + h}{R} \tag{1}$$

Where:
K_O is the scale factor due to the effect of height
R is the average radius
h is the average ellipsoidal height of the terrain.

TABLE 1. Cartographic and geodetic parameters of the LCPS

Zone	Local Central Meridian	Average height (m)	Ko
Zone 1	80º 09´ 30´´	59	1,0000092
Zone 2	79º 18´ 30´´	89	1,0000140
Zone 3	79º 14´ 00´´	2323	1,0003642
Zone 4	78º 32´ 00´´	2713	1,0004253
Zone 5	77º 53´ 00´´	2239	1,0003510
Zone 6	76º 57´ 30´´	857	1,0001343
Zone 7	76º 17´ 30´´	288	1,0000451

In order to demonstrate the decrease of the deformations produced by the LCPS with respect to the deformations produced by the UTM projection system, we used the model generated from the filling of the empty spaces of the SRTM elevation model with the level heights of the elevation model produced by the IGM. Such operation allowed to be performed as the precisions of both models are superior to the meter and for this reason the orthometric height may be considered similar to the leveled one (Carrión, 2013). The heights of the model were transformed to ellipsoidal using the geoid model EGM96 (Lemoine *et al.*, 1998). The bilinear method was used for its reliability when working with continuous data (Zhilin Li *et al.*,2005), ranging from a pixel size of 30 to 500 meters. This pixel size has been chosen as it is half of the maximum distance as being recommended to determine for topographic surveys scale of 1: 500 (Comisión Nacional de Riego, 2014). The model of the resampled elevation was evaluated with the GPS points of precision of the IGM, obtaining an ECM of 45.36 meters. This precision affects in the worst conditions at the level of the centimeter to the deformations.

The resampled model was converted into a shapefile format of point type in order to generate the mesh, where each point had the value of the height and the EAST coordinate that have been used to calculate the three factors that determine the deformation produced by each projective system. The linear anamorphosis factor was calculated using equation 2, which is an approximation of equation 3 that uses geodetic coordinates (Castillo, 2015; Millán, 2006). The value of the EAST coordinate and the scale factor due to the height effect depend on the area where the point is located, within the projective system. It needs to be emphasized that for the UTM projection system zones 17 and 18 we used a K_0 equal to 0.9996. The ellipsoidal height was used to calculate the reduction factor to the ellipsoid by 4, determining the deformation produced by the earth's relief. Finally, the combined factor was calculated with equation 5, which is an indicator of the difference between the distance measured in the cartographic plane and its corresponding in the field.

$$K_{esc} = K_o \cdot \left[1 + \left(\frac{x^2}{2 \cdot R^2}\right)\right] \quad (2)$$

$$K_{esc} = K_o \cdot \left[1 + \frac{\Delta\lambda^2}{2} \cdot cos^2\varphi \cdot \left(1 + cos^2\varphi \cdot \frac{e^2}{1 - e^2}\right)\right] \quad (3)$$

Where:
K_{esc} is the linear deformation module
x is the distance from the central meridian to the point (coordinate EAST - false EAST)
e it is the first eccentricity

$$K_h = 2 - \frac{R + h}{R} \quad (4)$$

Where:
K_h is the reduction factor to the ellipsoid

$$K_{comb} = K_h * K_{esc} \quad (5)$$

Where:
K_{comb} is the combined factor

The deformation models for each projection system have been generated from the combined factor and expressed in units of deformation, which represent the difference between the measurement in field with respect to its corresponding in the plane. The m/km was chosen as the deformation unit due to the ease with which the relative error is interpreted, where its calculation is based on equation 6. TABLE 2 lists a sample of the database created in order to generate the deformation model of the LCPS, where the constants are the average radius of the earth, the scale factor due to the height of each zone and the false EAST.

$$m/Km = (1 - K_{comb}) \cdot 1.000 \quad (6)$$

The database was first transformed into a shapefile format, where each point had its deformation value and subsequently it was transformed into a raster format in order to obtain a better visualization and interpretation of the behavior of the deformation. The size of the pixel established the distance of the separation between each point (500 meters). In addition, each pixel received the value of the point to which it belonged. In this way two deformation models were obtained.

For the determination of the compliance with the planimetric precision of each projective system, the m/km were transformed into error meters. This has been conducted by using the maximum distances that are recommended to be measured in order to perform topographic surveys for a scale of 1: 500 and 1: 1,000, which are 1 and 1.8 kilometers respectively (Comisión Nacional de Riego, 2014). These values were multiplied by the absolute value of the deformation of each pixel. In order to classify the generated raster model, the criterion

TABLE 2. Calculation of deformation factors

Height	East	X	Kh	Kesc	Kcomb	PPM	m/Km
25	500474,39	474,39	0,999996	1,000351	1,000347	-347,082	-0,347
50	540200,40	40200,40	0,999992	1,000371	1,000363	-363,024	-0,363
75	581989,77	81989,77	0,999988	1,000434	1,000422	-421,869	-0,422
3500	500474,39	474,39	0,999451	1,000351	0,999802	197,870	0,198
4000	744569,89	244569,89	0,999373	1,001086	1,000458	-458,498	-0,458
4500	830272,32	330272,32	0,999295	1,001692	1,000985	-985,176	-0,985

used was the tolerance established by the Military Geographic Institute. This has been obtained by multiplying the scale by the value of the smallest object that has been captured by human sight (geographic approximation of 0.3 mm). In this way we determined that the tolerance for scale 1: 500 is 0.15 meters and for scale 1: 1,000 is of about 0.30 meters (Instituto Geográfico Militar, 2006).

We generated four raster models, two for each projection system, where each model consists of two classes. The class of pixels that meet the planimetric precision, whose value for scale 1: 500 is equal to or less than 0.15 meters and it is represented with the green color and the pixels that fail, whose value is higher than 0.15 meters and is represented by the red color. The same classification procedure has been applied for the 1: 1,000 scale, where the tolerance value varied (0.30 meters). The minimum visual difference that may be observed in the areas of compliance is due to the fact that the tolerance between scales has a ratio of double (0.15 to 0.30 meters), while the value of the maximum distances that have been able to be measured in a topographic survey keeps almost the same relation (1 with 1.8 kilometers).

The compliance area of each scale for each projective system together with the statistical interpretation of the deformation, served to demonstrate the obtained improvement when using the LCPS to generate detailed cartography regarding the use of the UTM system. The statistical interpretation consisted of contrasting the deformation values of each pixel from the deformation models of each projection system, where the absolute mean, maximum and minimum, range and absolute deviation were calculated to determine which system had the least deformations. Only urban areas were considered, as in these places the highest precision cartography is generated. The statistical analysis of the deformation and compliance models was performed in the GIS.

3. Results and discussion

The comparison of the deformations produced by the use of the UTM system with respect to the developed LCPS, at the level of the continental territory, determined that there is a reduction about three times of the mean of the absolute value of the deformation. The maximum deformation varies from 0.688 to 0.556 (m/km), while the minimum deformation ranges from -0.923 to -0.499 (m/km). The aforementioned variations take relevance when comparing the range of the deformation between the projective systems, since it yields a variation of 1.611 (m/km) to 1.055 (m/km). Finally, the dispersion of the deformation improves since the standard deviation of the absolute value of the deformations varies from 0.179 to 0.087. All the aforementioned data are listed in TABLE 3, which indicates the improvement when using the developed LCPS in order to generate cartography in the continental territory.

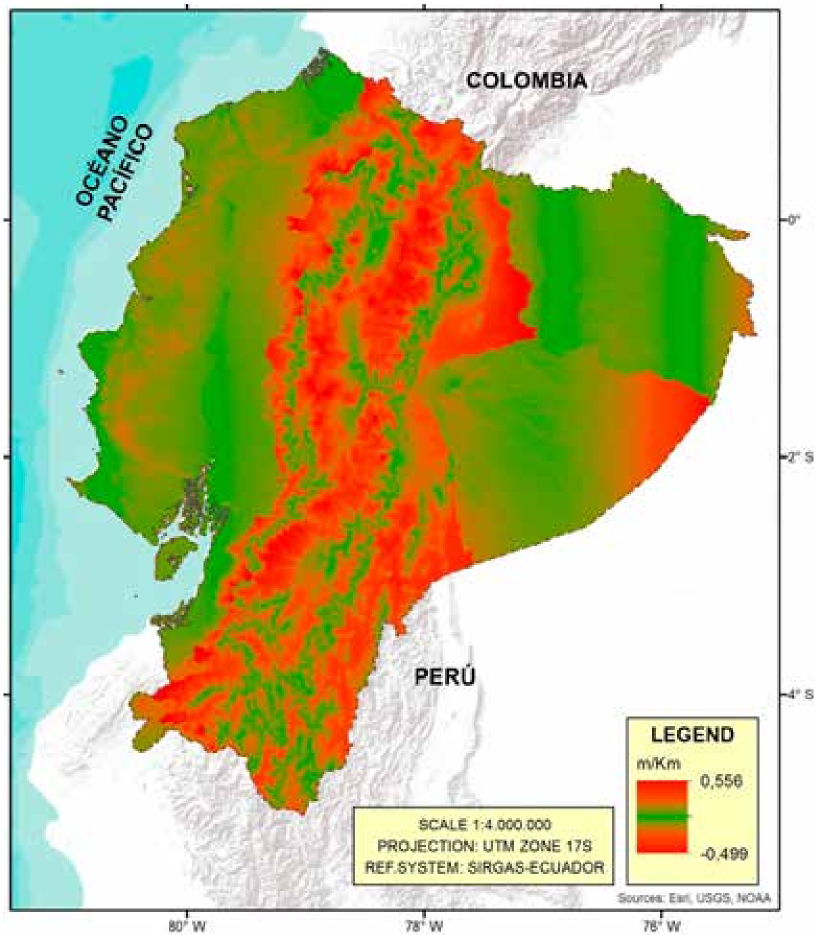
TABLE 3. Deformations obtained with the projection systems at the continental level

Deformation (m/Km)	UTM	LCPS
Mean (absolute)	0,265	0,096
Maximum deformation	0,688	0,556
Minimum deformation	-0,923	-0,499
Deformation range	1,611	1,055
Deviation (absolute)	0,178	0,090

The first model demonstrates the deformations that the UTM projection system presents (FIGURE 4), while the second model illustrates the generated deformations when using the LCPS (FIGURE 5).

At the level of the urban areas, the deformation behavior for the projective systems has been different. The mean of the absolute value of the deformations was reduced almost five times and the maximum deformation varied from 0.490 to 0.150 (m/km), while the minimum deformation

FIGURE 4. Deformation of the UTM projection system



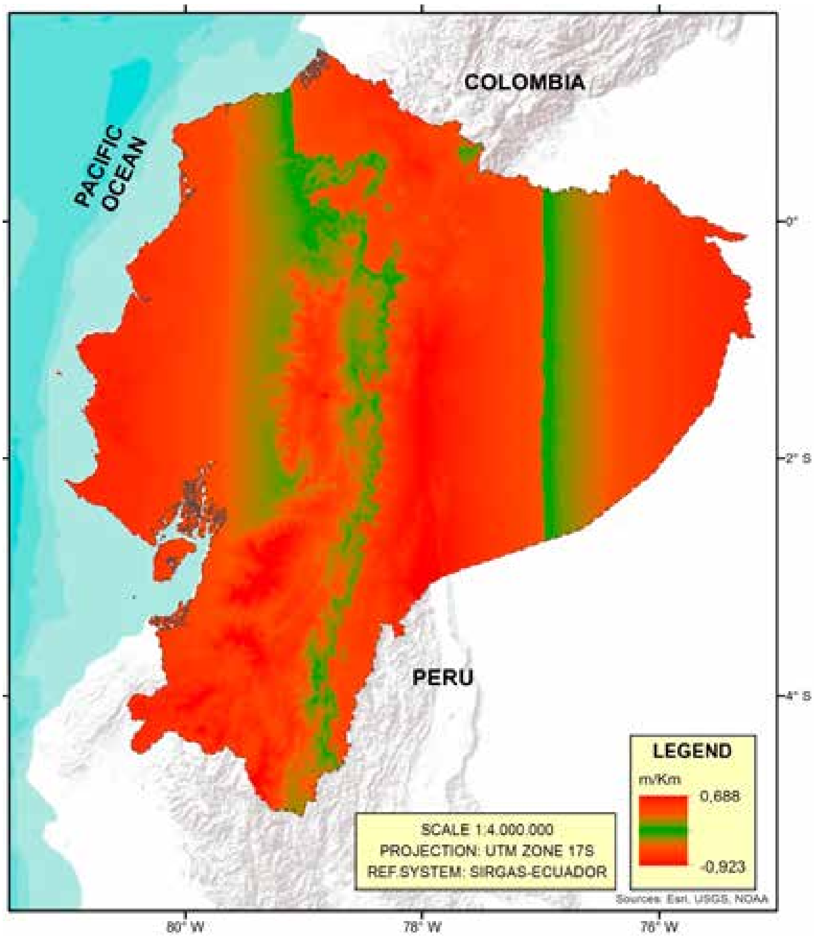
ranged from -0.820 to -0.403 (m/km). These values are even more remarkable when comparing the deformation range of the UTM projection system, which was 1,310 (m/km) with respect to that of the LCPS, which was 0.533 (m/km). Finally, the dispersion of the deformation improved since the standard deviation varied from 0.144 to 0.047. All the exposed values are listed in TABLE 4, which demonstrates the existing improvement when using the developed LCPS in order to generate cartography in the urban area.

TABLE 4. Deformations obtained with projection systems at the level of the urban area

Deformation (m/Km)	UTM	LCPS
Mean (absolute)	0,196	0,037
Maximum deformation	0,490	0,150
Minimum deformation	-0,820	-0,403
Deformation range	1,310	0,553
Deviation (absolute)	0,144	0,047

When comparing the compliance surfaces of the UTM projection system with respect to those of the LCPS, it has been determined that for the

FIGURE 5. Deformation of LCPS

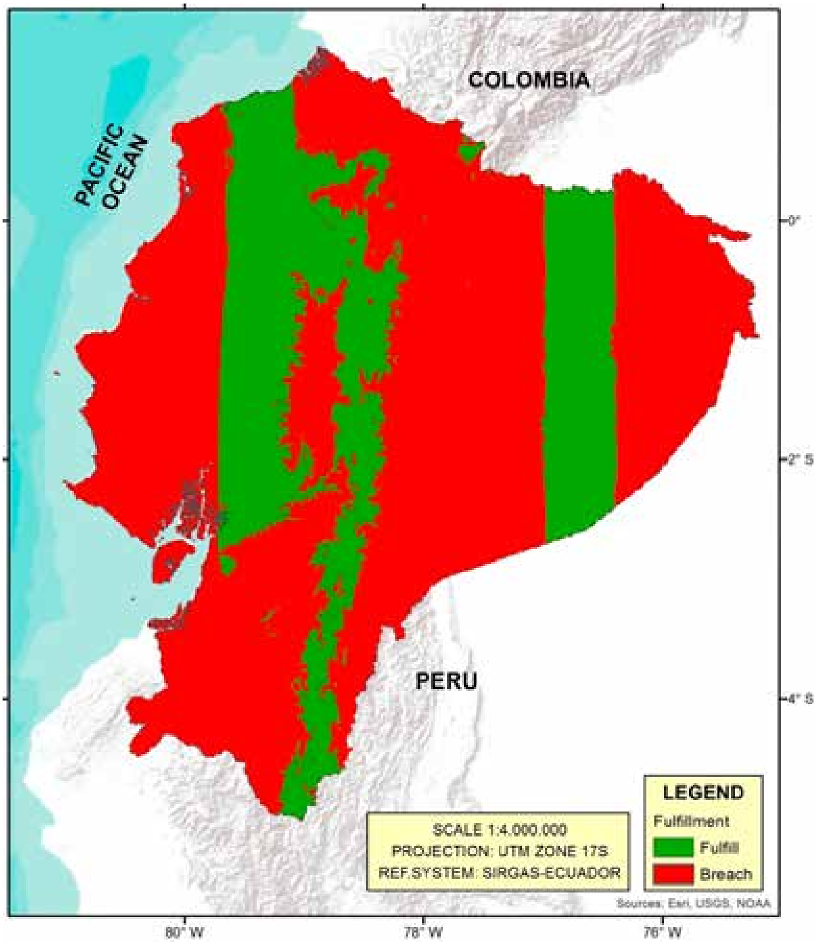


continental territory the compliance zone for scale 1:500 varied from 29.78% to 75.61%, while for scale 1:1,000 the percentage of compliance ranged from 32.81% to 78.89%. In both cases the surface area more than doubled. While for the urban area the compliance surface for scale 1: 500 varied from 41.39% to 96.95% and for scale 1: 1,000 the variation went from 43.27% to 97.33%, hereby maintaining the increase in more than double the compliance surface. This demonstrates that there is a greater area of territory, both of the continental territory and in the urban area that will comply with the planimetric precision when using the developed LCPS in order to generate detailed cartography.

Compliance with the planimetric precision of the UTM projection for scale 1: 500 is illustrated in FIGURE 6 and for scale 1: 1,000 in FIGURE 7. While compliance with the accuracy for the LCPS for scale 1: 500 is documented in FIGURE 8 and for scale 1: 1,000 in FIGURE 9.

As observed in the figure (deformation graph of the SPCL) there is an abrupt change in the deformations, this same phenomenon can be observed in the figures that represent the compliance

FIGURE 6. Compliance with the UTM projection system scale 1: 500



area of the SPCL system, where discontinuity is observed in the compliance and therefore, the deformations and compliance areas of each zone that are observed in TABLE 5 had to be analyzed.

The analysis of the deformation and area of compliance by the zone which conducts the LCPS is listed in TABLE 5, serving to a better understanding about the behavior of a variety of factors such as average height and area width (projection field). The result of an analysis of two extreme cases stand out. The first occurs in zone 2, which is characterized by being the least extensive area but with the greatest variation in height in all its extension, causing it to have the greatest variation in deformation and one of the better compliance area percentages with around 94%, demonstrating the weight of the zone width to control the compliance area. The conditions of the second case are presented in zone 6, where the largest extension of the projection field occur as well as the smallest variation in height. This caused the variation in deformation to be one of the lowest and a high percentage of the compliance area caused by the existence of a dominant amount

FIGURE 7. Compliance with the UTM projection system scale 1: 1,000

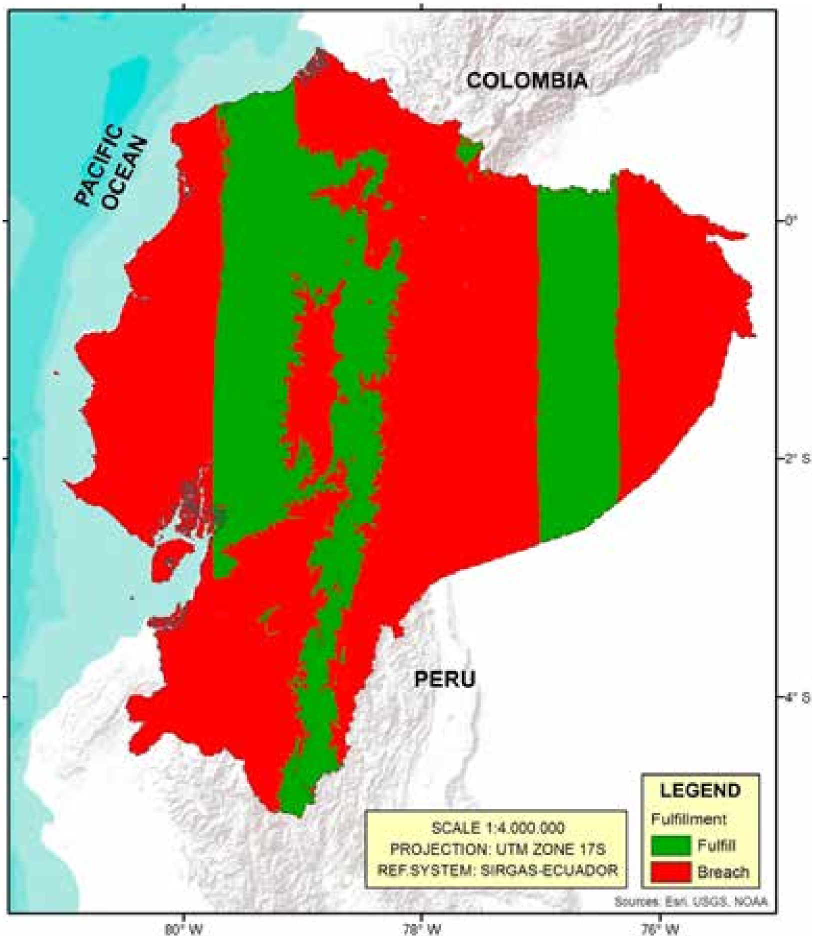
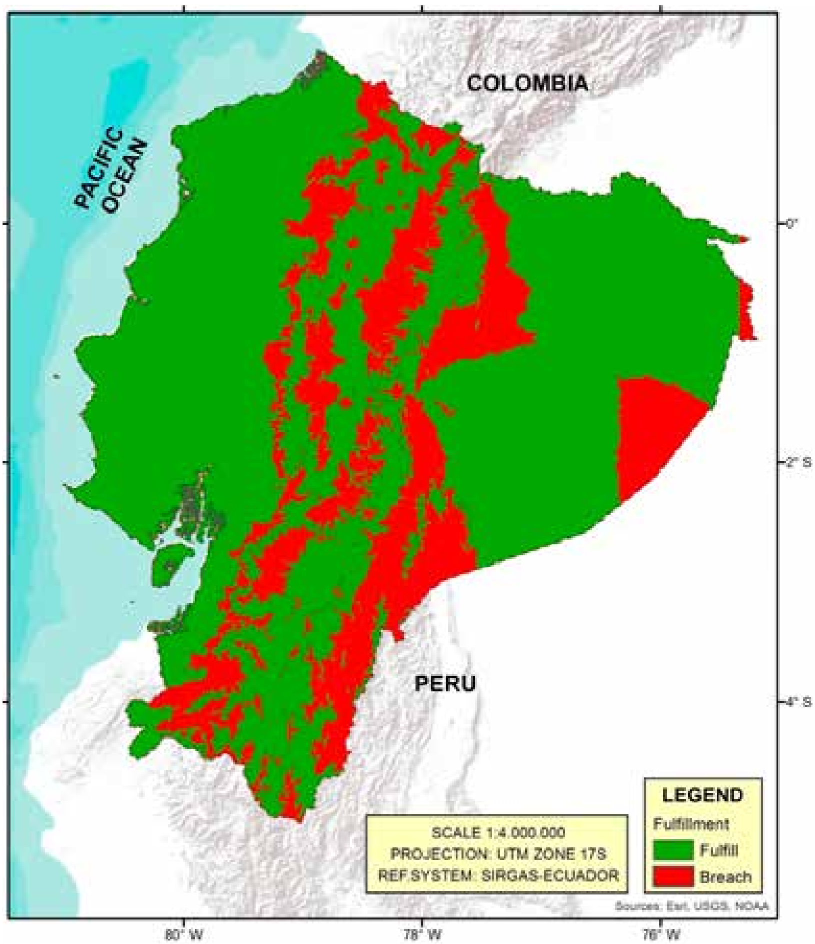


TABLE 5. Analysis of the deformation and compliance areas of the LCPS zones at continental Ecuador level

Zona	Zone width (°)	Δh in the whole area (m)	Deformation variation (m/Km)	Fulfillment (%)	Δh that meets the precision (m)
1	1,6	3218	-0,491 – 0.457	99,11	3216
2	0,9	3644	-0,409 – 0,556	94,43	3422
3	2,0	2794	-0,499 – 0.546	62,01	2261
4	1,2	3574	0,000 – 0,540	53,15	2383
5	1,8	3570	-0,407 – 0.553	38,74	2119
6	2,5	1082	-0,395 – 0.054	80,87	1082
7	2,4	2076	-0,411 – 0.118	98,13	2076

FIGURE 8. Compliance of the LCPS scale of 1: 500

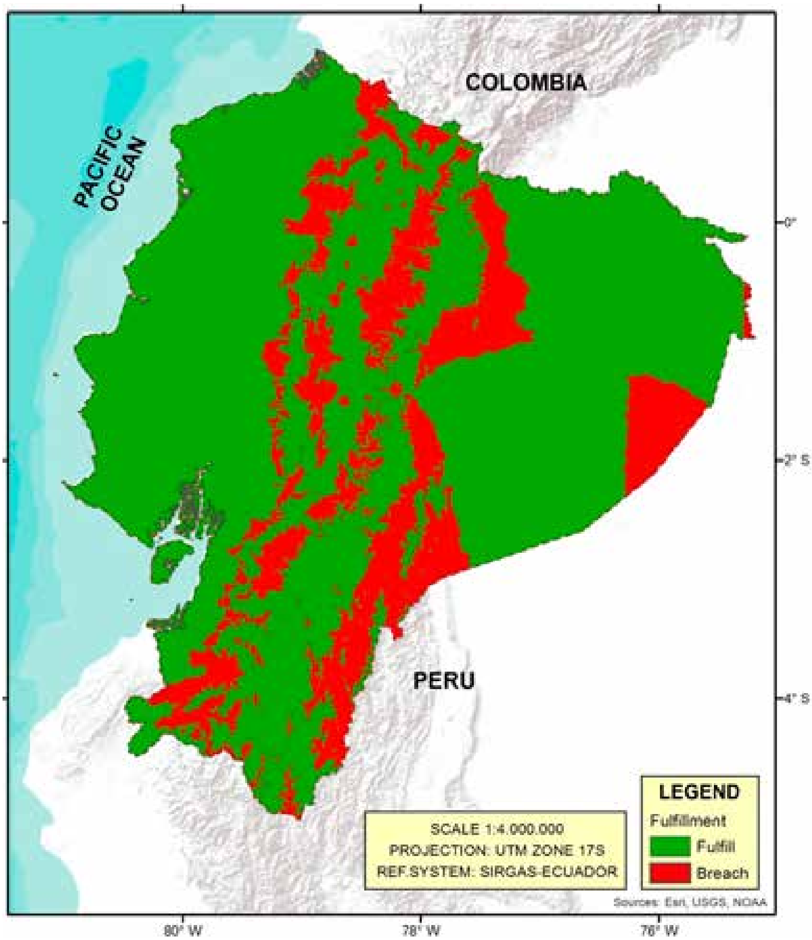


of heights near the average height defined for the area (857 meters), as illustrated in FIGURE 10.

Zone 7 has the second largest extension of the LCPS projection field and a considerable height variation. These conditions cause that the variation of the deformation is large and that the compliance area also is, but this last statement appears to be contradictory, since a high variation in height should cause a low percentage in the compliance area, as in zone 6. This does not occur as the distribution of heights for this area is even closer to the average height (288 meters) and better grouped, as illustrated in FIGURE 11. This highlights the importance of having a digital elevation model to define a LTM-PTL projection in a technically correct manner.

A concern may appear when analyzing FIGURE 10, about the chosen average height of 587 meters for zone 6. This may be clarified, as this choosing height is the purpose pursued by the LCPS, which is to ensure compliance with the accuracy of the cadastral cartography and it is the urban areas where the most accurate cartography is generated. In this way the deformation was analyzed at the

FIGURE 9. Compliance of the LCPS scale 1: 1,000



level of the urban area by the area of the LCPS, as based on the data listed in TABLE 6. There a reduction in the variation of the heights may be observed, which causes a reduction in the deformations. Zone 4 presents the greatest variation in height, but contrary to what would may be expected, there is a high percentage of compliance area. The importance of the distribution of heights, which are grouped near the average height (2713 meters), may be reaffirmed in FIGURE 12.

Zone 6 presents one of the lowest variations of the height and a low percentage of the area of compliance. This phenomenon may be explained when observing FIGURE 13, where it illustrates that the encountered urban areas are very dispersed within the geographical space together with a notable difference in height between each urban area and a very scattered distribution of heights (FIGURE 14), where it causes a low percentage in the area of compliance. All these facts allow to explain the selection of the average height for zone 6, documenting apparent complications in the determination of the average height, when there is a large variation in height in different urban areas.

TABLE 6. Analysis of the deformation and compliance areas of the LCPS zones at the level of the urban area

Zone	Δh in whole zone (m)	Deformation variation (m/Km)	Fulfillment (%)	Δh that meets the precision (m)
1	638	-0,079 – 0,081	100,00	638
2	699	-0,054 – 0,092	100,00	699
3	2155	-0,403 – 0,131	91,99	2012
4	2208	-0,378 – 0,134	97,75	949
5	1870	-0,279 – 0,150	74,79	996
6	411	-0,157 – -0,111	30,00	411
7	132	-0,108 – -0,006	100,00	132

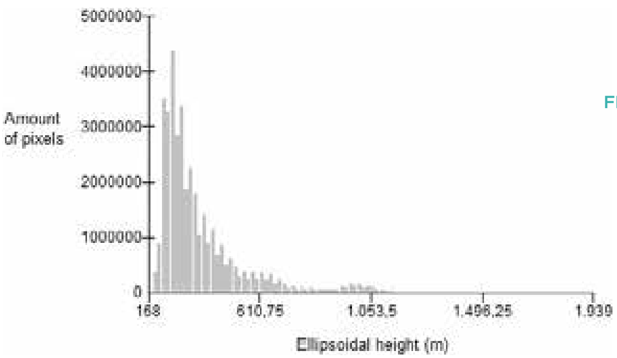


FIGURE 10. Distribution of heights in zone 6

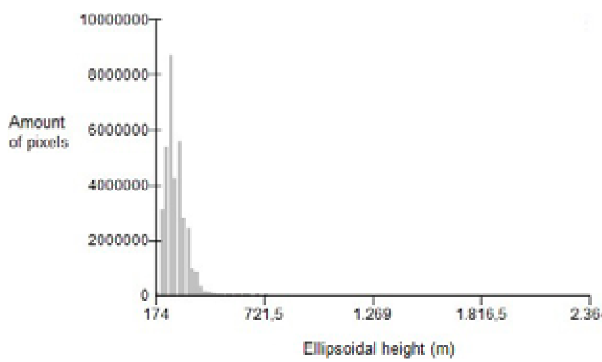


FIGURE 11. Distribution of heights in zone 7

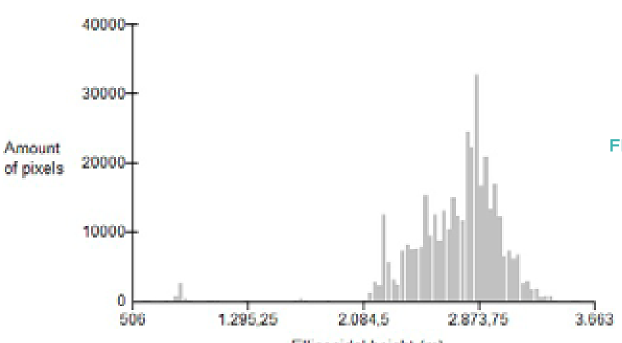


FIGURE 12. Distribution of heights in the urban area of zone 4

FIGURE 13. Location of the urban area in zone 6

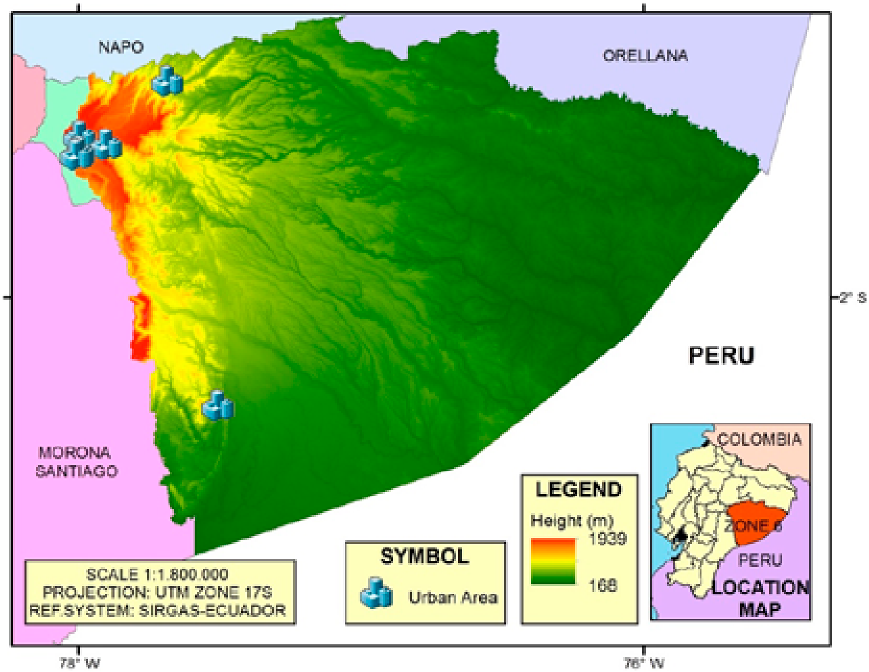
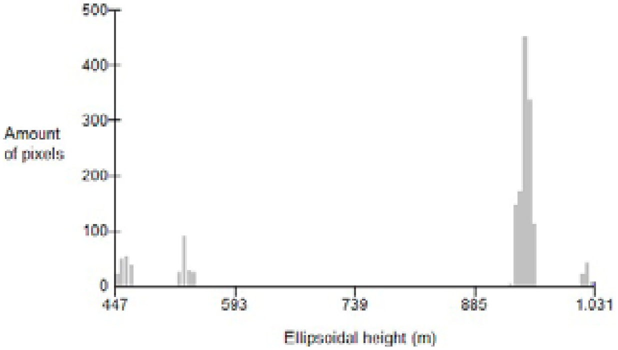


FIGURE 14. Distribution of heights in the urban area of zone 6



We compared our obtained results with other studies. A topographic survey of a section of the road between Puno and Tiquilla, in Peru, it has been demonstrated that the project, being located at an average ellipsoidal height of 4098.94 meters and 115 km away from the central meridian, the deformation produced by using the UTM projection has been of about 0.876 m/km (Aduviri, 2017). This result supports the values obtained in the deformation model of the UTM system generated for continental Ecuador. A further study documented that a variation of more than 2,000 meters to scale 1: 1,000 and 1,000 meters to scale 1:500, prevents compliance with the planimetric precision for each scale, since the deformations produced are 31.4 (cm/km) and 15.7 (cm/km) respectively (Sanchez, 2008), confirming the behavior of the deformation as listed in TABLES 5 and 6.

There are several countries that have detected the need to generate local projections that allow to meet the requirements of detailed cartography. In Sweden a TM projection system was generated to support the SWEREF 99, where the zones are characterized by having an extension of 3° and a scale factor in the central meridian equal to one, in order that the local topography complies with the precisions that it needs (Engberg *et al.*, 2002). While in Australia local point projections have been used, in several projects such as the Perth Coastal Network in Western Australia or the Integrated Survey Grid (ISG) covering New South Wales, where the central scale factor in the central meridian is adjusted in such a way that the grid distances correspond closely to the distances measured on the earth's surface (Featherstone *et al.*, 1999).

A further study in Chile has been used as a guide for the development of the LCPS, but there have been several differences, such as the number of 20 zones that have been established within Chilean territory, resulting to be the adequate established amount of zones in order to comply with the high precision required by topographic road surveys, this being higher than 25 ppm (Castillo, 2015). The projective system generated in the present study consisted of seven zones with which an average precision of 96 ppm has been reached for the continental territory and 37 ppm for the urban area. The increase in accuracy may be explained in terms of the purpose pursued the LCPS, complying with the requirements of cadastral cartography, which is less demanding than road mapping. With all the above, we have been able to demonstrate that the developed LCPS ensures a better planimetric precision and unifies the cadastral cartography generated at scales 1: 500 and 1: 1,000, in comparison with the UTM projection system.

5. Conclusions

The local projection system presented in this study for Continental Ecuador, has been divided in seven zones that indicate the limits for which this projection is ideal to represent the characteristics of the different regions to be mapped.

The percentage of compliance and the deformations for each zone are determined by the width of the zone (a projection field) and the average height. The latter having a greater influence on the distribution of the same. When the distribution of the heights is close to the average height, the compliance area for each zone increases, therefore, the zones that are within the Ecuadorian highlands presented a great limitation due to the notable variations of height, which prevented to fulfill the planimetric precision that the detailed cartography requires. This effect has been diminished, based to the fact that administrative units in this region have a narrower projection field. A possible solution to increase the percentage of compliance area and reduce deformations, is to define zones for each non-compliance surface, but this solution has not been applied as a large number of zones for the LCPS would hinder its application in practice, besides that there are areas that due to their high altitude and natural conditions would prevent the development of cities.

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Lugar y fecha de finalización del artículo:
Quito, Ecuador; julio, 2019